Find the derivative for each of the following functions.

1. \( f(x) = x^4 + 7x^3 - 2x^2 - 4x + 3 \),
2. \( h(t) = t^3 - 5t + \frac{1}{2} - \frac{1}{t^2} \),
3. \( p(z) = \sqrt[3]{z} + 4.7z^2 - 7\sqrt{z^5} \),
4. \( q(w) = 3w^{-0.4} + 2.1w^5 - \frac{2}{\sqrt{w}} \),
5. \( g(x) = A - \frac{B}{x^2} + \frac{C}{\sqrt{x}} - Dx^4 \).

6. In the linear section, we found that the growth of a child satisfies the equation

\[ h(a) = 6.46a + 72.3, \]

where the age, \( a \), is in years and the height, \( h \), is in cm.

a. Find \( \frac{dh}{da} \). What is the growth rate at age 2? At age 6?

b. If a child is 135 cm at age 10, what is the predicted height at age 11?

7. The number of species of herpatofauna, \( N \), on Caribbean Islands as a function of the area in square miles, \( A \), is approximated by the formula

\[ N = 3A^{\frac{1}{3}} \]

Find the rate of change in number of species as a function of area, \( \frac{dN}{dA} \), when the area of the island is 64, 125, and 1000 square miles. You should sketch a graph of the derivative, \( \frac{dN}{dA} \), for \( 0 \leq A \leq 1000 \).

8. A ball that is thrown vertically falling under the influence of gravity without air resistance from a 128 ft platform with an upward velocity of 32 ft/sec satisfies the equation

\[ h(t) = 128 + 32t - 16t^2, \]

where \( h \) is in feet and \( t \) is in seconds.

a. Find an expression for the velocity, \( v(t) = h'(t) \). Determine when the velocity is zero, then determine the maximum height of the ball. What is the velocity at \( t = 2 \) and \( t = 4 \)?

b. You should sketch a graph of \( h(t) \), showing crucial points, including the \( h \)-intercept, the maximum height, and determine when the ball hits the ground.

9. A cat is crouching on a ledge that is 12 feet above the ground, trying to ambush pigeons that fly by.

a. Suppose that a pigeon flies by 4 feet above the cat, and that the cat jumps off the ledge with just enough vertical velocity, \( V \) to catch the pigeon. If the height of the cat is given by

\[ h(t) = -16t^2 + Vt + 12, \]

then find the velocity \( v(t) = h'(t) \) of the cat at any time, \( t \geq 0 \).
b. Find when the velocity is equal to zero in terms of $V$. This is the time at the maximum height.

c. Since the cat is 16 ft in the air at this time, use the equation for the height of the cat, $h(t)$, to compute the initial velocity of the cat, $V$. Substitute this into the velocity equation, $v(t)$, to give the velocity of the cat after 1 second.

d. Find when the cat hits the ground with the pigeon and find the velocity of the cat the instant before it hits the ground.

10. Suppose that a population of insects, $P$ (individuals), in a controlled experiment with constant food supply is shown to have a growth rate that fits the logistic growth function

$$g(P) = 0.04P \left(1 - \frac{P}{800}\right),$$

where the time is measured in days.

a. Find the population when the growth rate $g(P)$ is zero (the $P$-intercepts). This gives the equilibria for this experiment. Think about a biological interpretation for each of the equilibria.

b. Compute the derivative of $g(P)$, then determine the population where $g(P)$ has a maximum growth rate, and what the maximum growth rate is. You should think about the biological interpretation of this maximum growth rate, and sketch the graph of $g(P)$.

11. a. Lizards are cold-blooded animals whose temperatures roughly match the surrounding environment. Suppose the body temperature, $T(t)$, of a lizard is measured for a period of 18 hours from midnight until 6 PM. The body temperature (in °C) of the lizard over this period of time (in hours) is found to be well approximated by the polynomial

$$T(t) = -0.01t^3 + 0.285t^2 - 1.80t + 15.$$

Find the general expression for the rate of change of body temperature per hour $\left(\frac{dT}{dt}\right)$.

b. Use this information to find what the rate of change of body temperature is at midnight, 4 AM, 8 AM, noon, and 4 PM. Which of these times gives the fastest increase in the body temperature and which shows the most rapid cooling of the lizard?