1. Consider $y = 15 + 2x - x^2$, then the first derivative is $y'(x) = 2 - 2x$ and the second derivative is $y''(x) = -2$. The $y$-intercept satisfies $y(0) = 15$ or $(0, 15)$. The $x$-intercepts satisfy $15 + 2x - x^2 = (5 - x)(3 + x) = 0$, so $x = -3$ or $x = 5$ The critical point occurs where $y'(x) = 2 - 2x = 0$, which implies $x_c = 1$ and $y(x_c) = 16$. Since $y''(x_c) = -2 < 0$, this is a maximum. The graph appears below to the left.

![Problem 1](image1.png)

2. Consider $y = x^3 - 12x$, then the first derivative is $y'(x) = 3x^2 - 12$ and the second derivative is $y''(x) = 6x$. The $y$-intercept satisfies $y(0) = 0$ or $(0, 0)$. The $x$-intercepts satisfy $y = x^3 - 12x = x(x^2 - 12) = 0$, so $x = 0, \pm 2\sqrt{3}$. Critical points occur where $y'(x) = 3x^2 - 12 = 0$, so $3(x_c - 2)(x_c + 2) = 0$. This implies $x_{1c} = -2$ and $y(x_{1c}) = (-2)^3 - 12(-2) = 16$. Since $y''(x_{1c}) = -12 < 0$, this is a maximum. Also, $x_{2c} = 2$ and $y(x_{2c}) = (2)^3 - 12(2) = -16$. Since $y''(x_{2c}) = 12 > 0$, this is a minimum. There is a point of inflection where $y''(x_p) = 6x = 0$. Therefore $x_p = 0$ and $y(x_p) = 0$. The graph appears above to the right.

![Problem 2](image2.png)

3. Consider $y = x^4 - 2x^2 + 1$, then the first derivative is $y'(x) = 4x^3 - 4x = 4x^3 - 4x$ and the second derivative is $y''(x) = 3 \cdot 4x^2 - 4 = 4(3x^2 - 1)$. The $y$-intercept satisfies $y(0) = 1$ or $(0, 1)$. The $x$-intercepts satisfy $y = x^4 - 2x^2 + 1 = (x^2 - 1)(x^2 - 1) = (x+1)(x-1)(x+1)(x-1) = 0$, so $x = \pm 1$. Critical points occur where $y'(x) = 4x^3 - 4x = 0$, so $4x(x^2 - 1) = 0$ so $x_c = 0, \pm 1$. When $x_{1c} = -1$, then $y(x_{1c}) = (-1)^4 - 2(-1)^2 + 1 = 0$. $y''(x_{1c}) > 0$, so this is a minimum. Also, when $x_{2c} = 0$, then $y(x_{2c}) = (0)^4 - 2(0)^2 + 1 = 1$. $y''(x_{2c}) < 0$, so this is a maximum. When $x_{3c} = 1$, then $y(x_{3c}) = (1)^4 - 2(1)^2 + 1 = 0$. $y''(x_{3c}) > 0$, so this is a minimum. There is a point of inflection where $y''(x_p) = 4(3x^2 - 1) = 0$. Therefore, $x_p = \pm \frac{1}{\sqrt{3}}$ with $y \left( \pm \frac{1}{\sqrt{3}} \right) = \frac{1}{9} - \frac{2}{3} + 1 = \frac{4}{9}$. The graph appears below to the left.
4. Consider \( y = 2x + \frac{2}{x} \), then the first derivative is \( y'(x) = 2 - 2x^{-2} = \frac{2x^2 - 2}{x^2} \) and the second derivative is \( y''(x) = -2 \cdot -2x^{-3} = \frac{4}{x^3} \). Since there is a vertical asymptote at \( x = 0 \), there is no \( y \)-intercept. The \( x \)-intercepts would satisfy \( y = 2x + \frac{2}{x} = \frac{2x^2 + 2}{x} = 0 \), which has no real solution. Thus, there are no \( x \)-intercepts. There are no horizontal asymptotes. Critical points occur where \( y'(x) = 0 \), or \( 2x^2 - 2 = 2(x + 1)(x - 1) = 0 \), so \( x_c = \pm 1 \). When \( x_{1c} = -1 \), then \( y(x_{1c}) = 2(-1) + \frac{2}{-1} = -4. y''(x_{1c}) < 0 \), so this is a maximum. Also, when \( x_{2c} = 1 \), then \( y(x_{2c}) = 2(1) + \frac{2}{1} = 4. y''(x_{2c}) > 0 \), so this is a minimum. The graph appears above to the right.

5. a. Since \( T(t) = 0.002(t^3 - 45t^2 + 600t + 16000) \), the derivative is \( T'(t) = 0.002(3t^2 - 90t + 600) = 0.006t^2 - 0.18t + 1.2 \). At noon, \( T'(12) = -0.096^\circ \text{C}/\text{hr} \).

b. The derivative can be written:
\[
\frac{dT}{dt} = 0.006(t^2 - 30t + 200) = 0.006(t - 10)(t - 20).
\]
Thus, critical values of \( t \) are \( t = 10 \) and \( t = 20 \). It is easy to see from the graph that \( t = 10 \) corresponds to a maximum and \( t = 20 \) corresponds to a minimum. The maximum temperature of the subject occurs at 10 a.m. with a temperature of 37\(^\circ\text{C} \), while the minimum temperature of the subject occurs at 8 p.m. \( (t = 20) \) with a temperature of 36\(^\circ\text{C} \). The graph appears below to the left.
6. a. Since \( Y(t) = \frac{1}{3}t^3 - 6t^2 + 20t + 120 \), the derivative is given by \( Y'(t) = t^2 - 6 \cdot 2t + 20 = t^2 - 12t + 20 \). The rate of change in oxygen consumption at \( t = 6 \) is \( Y'(6) = -16 \mu l/hr/hr \).

b. There are critical points when \( Y'(t) = t^2 - 12t + 20 = (t - 2)(t - 10) = 0 \), so \( t_c = 2 \) and 10. There is a maximum at \( t = 2 \) with \( Y(2) = \frac{416}{3} \) (substituting in the original equation). There is a minimum at \( t = 10 \) with \( Y(10) = \frac{160}{3} \). The graph appears above to the right.

c. The \( O_2 \) consumptions at the beginning and end are \( Y(0) = 120 \) and \( Y(12) = 72 \). The relative maxima and minima found in (b) are the absolute maxima and minima.

7. a. Consider the function \( T(t) = 0.01(1600 - 135t + 27t^2 - t^3) \). The derivative is given by \( T'(t) = 0.01(-135 + 54t - 3t^2) \). The rate of change of temperature per hour at \( t = 3 \) is \( T'(3) = 0 \).

b. There are critical points when
\[
T'(t) = -0.03(t^2 - 18t + 45) = -0.03(t - 3)(t - 15) = 0,
\]
so \( t_c = 3 \) and 15. There is a minimum at \( t = 3 \) with \( T(3) = 14.11 \). There is a maximum at \( t = 15 \) with \( T(15) = 22.75 \).

c. The temperatures at the beginning and end of the study are \( T(0) = 16 \) and \( T(20) = 17 \). The relative maximum and minimum are the same as the absolute maximum and minimum. The graph appears below.

8. a. The height of the impala satisfies \( h(t) = Vt - 490t^2 \). Differentiating to obtain the velocity, we have \( v(t) = h'(t) = V - 980t \).

b. \( v(t) = 0 \), when \( t_{\text{max}} = \frac{V}{980} \). We substitute this value into the original height equation to obtain:
\[
h(t_{\text{max}}) = V \left( \frac{V}{980} \right) - 490 \left( \frac{V}{980} \right)^2 = \frac{V^2}{1960} = 180.
\]
It follows that \( V^2 = 180(1960) \), so the initial velocity to clear the fence is \( V = 420\sqrt{2} \approx 593.97 \text{ cm/sec} \).
c. By symmetry of the parabola, the time to the \( t \)-intercept is twice the time to the maximum. The hang time is 
\[
t = 2t_{\text{max}} = \frac{\frac{V}{390}}{\frac{6}{7} \sqrt{2}} \approx 1.212 \text{ sec.}
\]