Calculus for the Life Sciences II
Lecture Notes – Introduction

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Outline

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   - TA Contact Information, Office Hours

2. The Class — Overview
   - Syllabus
   - Grading
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   - Computer Lab
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4. Introduction
   - Why Math 122 is needed for Biologists
   - Mathematical Models
   - Example – Predator-Prey Model
   - Example – Muscle Contraction Model
## Contact Information

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<td>Office Hours</td>
<td>1-2 MW and 3-4 MW, and by appointment</td>
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# TA Contact Information

<table>
<thead>
<tr>
<th>TA</th>
<th>Vinnie Berardi</th>
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<td>Office Hours</td>
<td>1:30-3 TTh in GMCS 425, and by appointment</td>
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</tbody>
</table>
Basic Information: The Book

**Title:**
“Calculus: A Modeling Approach for the Life Sciences”
Volume II

**Authors:**
Joseph M. Mahaffy & Alexandra Chávez-Ross

**Publisher:**
Pearson Custom Publishing

**ISBN:**
0-536-90522-3
Basic Information: Syllabus

- Review Derivative
- Discrete Dynamical Models
- Optimization
- Trigonometric Functions
- Differential Equations and Integration
  - Linear Differential Equations
  - Numerical Differential Equations
  - Integration
  - Separable Differential Equations
  - Integration by Substitution
  - Riemann Sums/Numerical Integration
  - Definite Integral
  - Integration by Parts
  - Qualitative Analysis of Differential Equations
Basic Information: Grading

Detailed information is found on the Homework and Assignment Web Page
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- **Lecture Material is 2/3 of grade**
  - Homework with WeBWorK (20% of Lecture grade)
  - 3 Exams (16% each)
  - Final (32%)

- **Scientific Calculator only** - Exams and Final
  - One 3x5 notecard for Exams and three 3x5 notecards for Final
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- **Lab Work is 1/3 of grade**
  - 8-10 Lab assignments
  - 3 Lab Exams worth twice a regular Lab assignment
    - Open notes, Flashdrive, **no email or cell phone**
    - Use Laptop or assigned computer in GMCS 425 or 422
Most class attendance is OPTIONAL — Homework and announcements will be posted on the class web page. If/when you attend class:
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- Please turn off mobile phones.
- Please be courteous to other students and the instructor.
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If/when you attend class:

- Please be on time.
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- Please turn off mobile phones.
- Please be courteous to other students and the instructor.
- Abide by university statutes, and all applicable local, state, and federal laws.
• WeBWorK assignments are posted with a specific due date. It is your responsibility to complete the assignment on time.
Expectations and Procedures, II

- WeBWorK assignments are posted with a specific due date. It is **your responsibility** to complete the assignment on time.

- The instructor will make special arrangements for students with documented learning disabilities and will **try** to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. **Please contact the instructor EARLY regarding special circumstances.**
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- Students are expected and encouraged to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!
Expectations and Procedures, III

- **Missed Exams or Lab Exams: Don’t miss Exams!**
  You will receive a **ZERO** for any missed exam, except for **written/documentated** excuses (illness, personal/family crises, etc.).
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Lab assignments:
- Attendance is mandatory or automatic 10 point deduction
- Partners are assigned and must work with given partner
- Arriving 20 minutes late or missing a Lab means working the lab alone
- Labs due promptly by Saturday 4 PM following a given Lab unless told otherwise.
- Lowest lab score is dropped
- Your responsibility to back up Lab work – No excuses accepted or extensions granted for lost material
Computer Lab

- Computer Labs are located in GMCS 422 and 425 – Hours are posted on the Lab doors
- Completed Lab Reports are turned into Math 122 box located in GMCS 425
- Software used
  - Excel
  - Word
  - Maple
- Labs are 60% WeBWorK and 40% written report
- **Please direct questions first to your Lab TA**
Math 122: Formal Prerequisites

Successful Completion of Math 121
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Successful Completion of Math 121

Warning! – If your Math 121 relied heavily on Wolfram Alpha, then you are at a very distinct disadvantage in this class.
Math 122: Introduction

- Biology is rapidly expanding - more quantitative analysis of the data
- Mathematics and computers have an increasing role in biology
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  - Emphasis on mathematical modeling of biological systems
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  - Begin with a biological model
  - Mathematical theory required to analyze the biological problem
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  - Lecture notes show how Calculus naturally arises in biological examples
  - Begin with a biological model
  - Mathematical theory required to analyze the biological problem
- Use real or realistic examples
- Computer labs aid solving more complicated models
Mathematical Biology

- Mathematical tools
  - Better qualitative and quantitative understanding of biological problems
  - Suggest alternate possibilities
  - Reject inconsistent ideas
Mathematical Biology

- **Mathematical tools**
  - Better qualitative and quantitative understanding of biological problems
  - Suggest alternate possibilities
  - Reject inconsistent ideas

- **Biological problems**
  - Often stretch mathematical techniques
  - Illustrate mathematical tools well
  - Build intuition for problem solving techniques
So what is a mathematical model?

- Real World
- Mathematical Model
- Empirical Data

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Mathematical Modeling

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- It exhibits the basic properties of the real system
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- The model should be testable against empirical data
- Comparisons of the model to the real system should lead to improved mathematical models
- The model may suggest improved experiments
Predator-Prey Model

Thanks to Tom and Pat Leeson
In the early 20th century, Sir Ronald Ross used mathematical modeling to show that malaria could be eliminated without the total eradication of mosquitoes.
Example – Predator-Prey Model

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- Widely used by biologists – however, significant flaws in the mathematical understanding often lead to poor conclusions.

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Classic Lynx-Hare Data

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- We’ll examine this model late in the semester.
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- Rapid rise in the population of the hares is followed by a rapid rise in the lynx population.
- Next the hare population plummets, which is followed by lynx population plummeting.
Muscle Contraction

- Muscles take chemical energy in our bodies and convert it into motion
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- These filaments slide past each other during muscular contraction due to cross-bridges between the two filaments
- The force of contracting muscles is generated by chemical reactions that cause a conformational strain in the head of the cross-bridges
Example 2 – Muscle Contraction

The figures show a single cross-bridge projection coming from the myosin filament and indicates how the motion of the head of this cross-bridge element can pull the actin filament toward the center of the cell.
Ratchet Theory of Muscle Contraction

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- Biochemical details have been intensely studied

Cross-bridge Cycle - Biochemical Steps

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- The ATP is hydrolyzed to ADP, which results in a conformational change in the head of the cross-bridge and generates the force to pull the actin filament toward the center of the cell.
- Next the ADP is released and binding between the filaments is broken, and the cycle is ready to begin anew with the next binding of ATP.
Example 2 – Muscle Contraction

Modeling Process

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- The process of understanding contraction of a muscle relies on collaborations between biologists studying the muscle tissue, physicists and mathematicians who can calculate the forces generated and required configurations, and chemists who determine key atomic structures, which require massive computational methods.
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- A mathematical model for the sliding filament theory is too complicated for this course, but we will be developing some of the basics, such as stretching of an elastic element, which are needed for modeling molecular action of the cross-bridges.