Calculus for Biology

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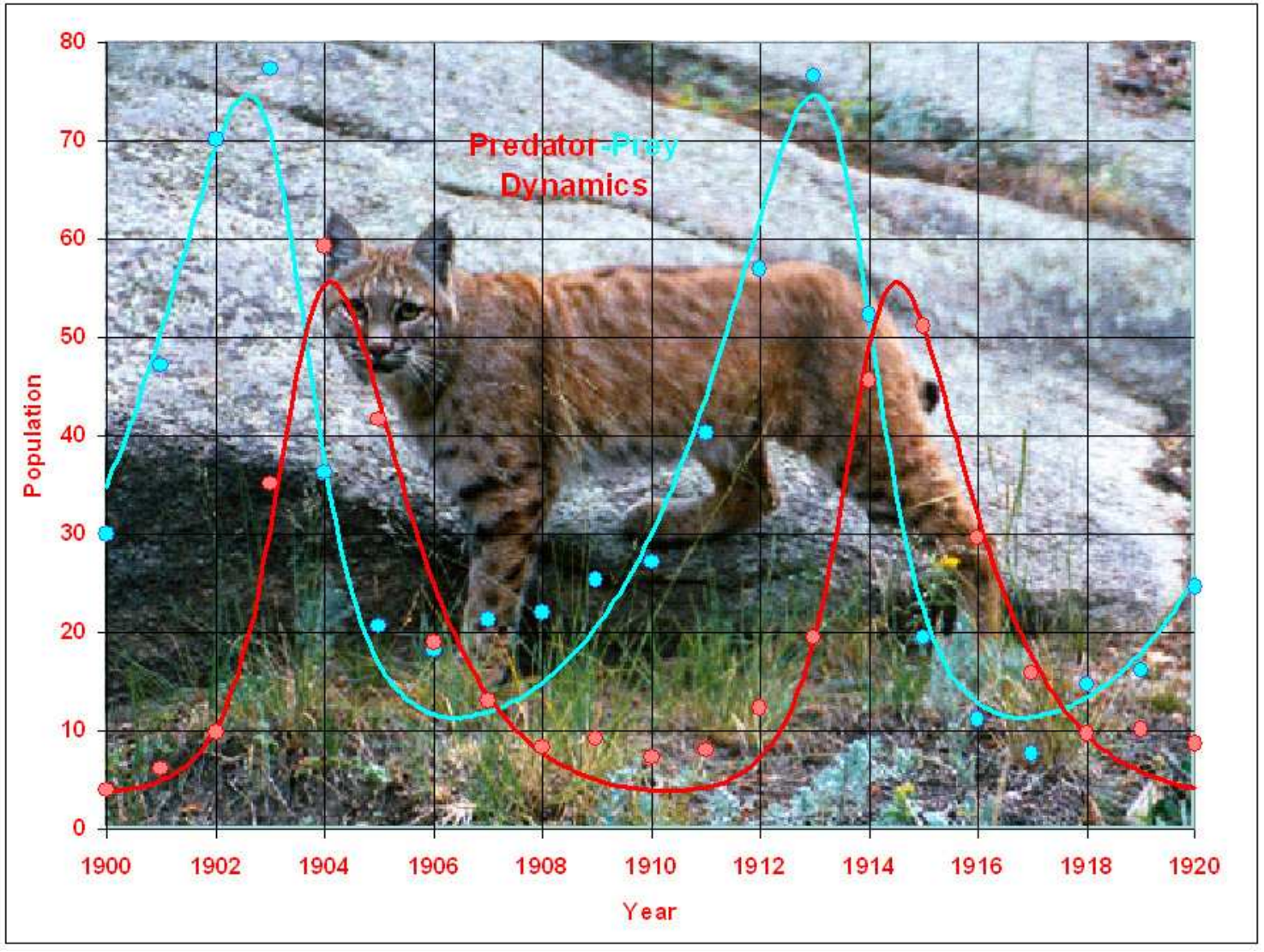
August 2007



Text available from Prentice Hall Custom publishing
Lab Manual available from Montezuma publishing (SDSU)

Calculus at San Diego State University

San Diego State University has approximately 35,000 students

- Calculus - Main Sequence

  3 semesters - 13 Units

- Business Calculus

  1 semester - 3 Units

- Calculus for Biology

  2 semesters - 6 units

Calculus for Biology - Background

- Students
  - Over 95% are Biology majors
  - Wide variety of backgrounds - High school calculus down to 8 year gap in any math courses
  - Most complain that math is no value to them
- Before late 1980s taught watered down calculus (using Business Calculus text)
- SDSU Math faculty claimed these were weak students, yet some of the best students become biology majors
- This allowed me to design a relevant course as others weren't interested
- Most students realize relevance after my courses!

Calculus Course

- 2 lecture (1 hr each) and 1 computer lab (2 hr)
  - About 150-250 students Math 121 each semester
  - About 75-100 students Math 122 each semester
- Modeling and dynamical systems approach
- Computer Lab
  - Uses mostly Excel and Word with some Maple
  - Most problems are biological that need computers to solve
  - Learn both graphing and writing skills
- Automated homework (WebWork)
- Web access to most material (See my website)

Web Access to Material

- Easiest access is Google Mahaffy

  Select Courses button

  Choose from Math 121 or Math 122

- Lecture style notes for projection

  Change index.html to index1.html

  At present, must use Internet Explorer for lecture-style notes

Show how website appears

Data and Calculus

Project NExT wants demonstrations of how their fellows could use Biological examples in Calculus (and other lower division classes). I will show a series of examples and give my philosophy for teaching Calculus.

Using Data

- Students prefer real examples
- Actual data is convincing though numbers may be messy
- Biological examples give easy connections for students
- Show **least squares fit to data** very early

Juvenile Height

- Examine data on juvenile height
- Show least squares fit to data - almost linear
- Ask modeling questions
  - Domain
  - Extrapolation
  - Asymptotes
  - Rate of growth

Show Applet in Least Squares section

mRNA Production

For my student audience, I lead with a biological example to motivate the students to learn the material

- Linear least squares fit to mRNA production as a function of growth rate

  Hard to find good biological examples that are straight lines through the origin
- Detailed biological description of the biological phenomenon provided on my website
- Least squares fit through origin is parabolic function of slope
- Connect minimum of quadratic to best line fitting the data
- Single variable version of general best line fit - multivariable calculus

Show Example and Applet in Quadratic section

Prozac

Use a significant example that most students relate to

- Drug metabolism is important application similar to radioactive decay
- Active metabolite often confounds drug studies
- Metabolite combines two exponentials with maximum
- Example easily modified for cascade systems in differential equations
- Most common application in pharmakinetics

Show graph of Fluoxetine and Norfluoxetine in exponential section

Dynamical Systems Approach

- Use Discrete Malthusian growth to motivate derivative

  Connects well to principle and interest problems from high school

  Growth is a derivative idea easy to understand (See juvenile height problem earlier)

- Other nonlinear discrete dynamical models

  Connect to discrete growth

  Equilibrium easy to understand

  Derivative gives stability of equilibrium is a good application

- Differential equations motivate the integral

  Linear differential equations follow easily from discrete dynamical models

  Easy to connect to growth laws

  Inverting leads naturally to antiderivatives/integrals

WebWork

- Developed problems for most exercises on Web page
- Students performed 10% better on exams with WebWork
- Gives immediate feedback

  Students bring problems of understanding immediately back to lecture
- Forces students to do homework

  Individualized problems

  Weight homework as midterm exam
- Easy to learn to create WebWork problems

Computer Labs

- Most computer labs designed to enhance lectures
- Hands on experience - Highly interactive
- Work in pairs - Change each time
- Develop skills with Excel, Maple, and writing (Word Lab reports)
- Allow students to work on real world problems - Processing data

Optimization

- Lecture examines Northwestern Crows dropping Whelks
  - Interesting set of data collected
  - Examine energy function
  - Show that data supports minimum energy expenditure

- Lab examines Glaucous-winged Gulls dropping Butter Clams
  - Students process data - Derive functions to use
  - Examine energy function, as lecture
  - Data **doesn't** match minimum energy, ask students differences between scenarios
  - Kleptoparasitism - Shows how modeling leads to new hypotheses

Show optimization lecture notes, seagull computer lab (with solution). Give brief Excel demonstration with Solver.

Growth of Fish

Allometric Modeling and Chain Rule

- Data fits von Bertalanffy equation for length depending on age

$$L(t) = L_0(1 - e^{-bt})$$

- Data on weight connected to length by allometric model

$$W = AL^k$$

- Chain rule gives dW/dt
- Second derivative gives point of inflection, which is when fish is most rapidly gaining weight - optimal harvesting

Show computer lab on growth of Pacific fish

Other Sources

- Website of John Jungck at Beloit College

  BioQuest

  ESTEEM

- Website of Lou Gross at University of Tennessee