

Computer Problem 2

(24pts) Consider the function $f(x) = x^2$.

1. The Fourier sine series satisfies:

$$f(x) = \begin{cases} x^2, & 0 \leq x < 3, \\ -x^2, & -3 < x \leq 0. \end{cases}$$

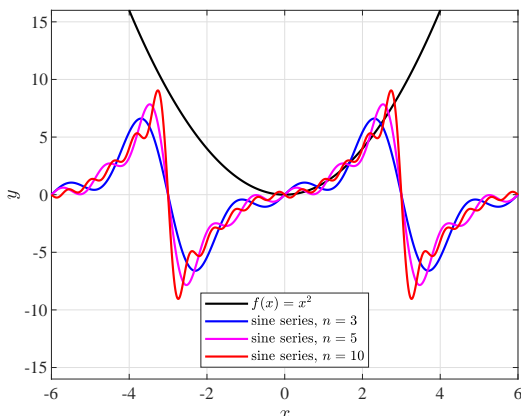
with $f(x)$ converging to 0 at $x = \pm 3$ and being a period 6 extension of this function. The Fourier sine series is given by:

$$f(x) \sim \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{3}\right),$$

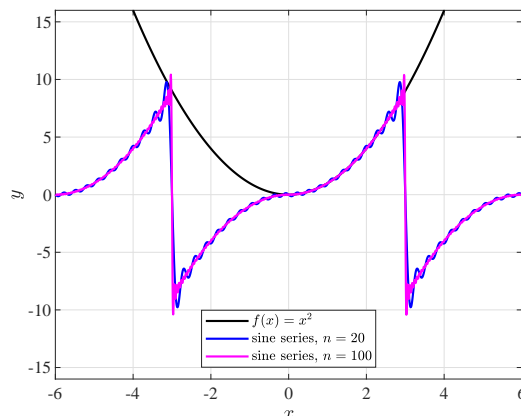
where the Fourier coefficients satisfy:

$$\begin{aligned} b_n &= \frac{2}{3} \int_0^3 x^2 \sin\left(\frac{n\pi x}{3}\right) dx, \\ &= -\frac{2x^2 \cos\left(\frac{n\pi x}{3}\right)}{n\pi} + \frac{12x \sin\left(\frac{n\pi x}{3}\right)}{n^2\pi^2} + \frac{36 \cos\left(\frac{n\pi x}{3}\right)}{n^3\pi^3} \Bigg|_0^3, \\ &= -\frac{18 \cos(n\pi)}{n\pi} + \frac{36 \cos(n\pi)}{n^3\pi^3} + \frac{18}{n\pi} - \frac{36}{n^3\pi^3}, \\ &= \left(\frac{18}{n^3\pi^3}\right) ((2 - n^2\pi^2)(-1)^n - 2). \end{aligned}$$

2. Below are graphs for $f(x)$ with $x \in [-6, 6]$ along with the Fourier sine series with 3, 5, 10, 20, and 100 terms.



$f(x)$ and Fourier sine with $n = 3, 5,$ and 10



$f(x)$ and Fourier sine with $n = 20$ and 100

3. The Fourier cosine series satisfies $f(x) = x^2$ for $x \in [-3, 3]$ with $f(x)$ converging at all values of x to a period 6 extension of this function. The Fourier cosine series is given by:

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{3}\right),$$

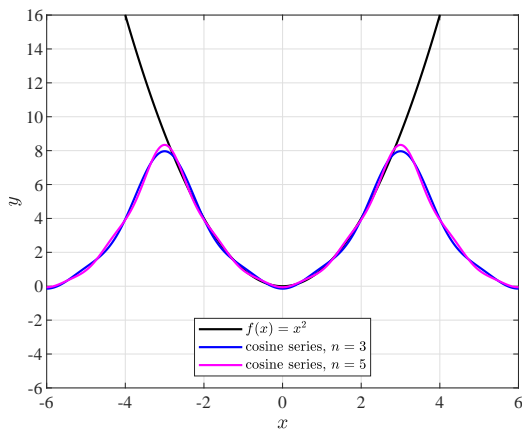
where the Fourier coefficients satisfy:

$$a_0 = \frac{1}{3} \int_0^3 x^2 dx = 3.$$

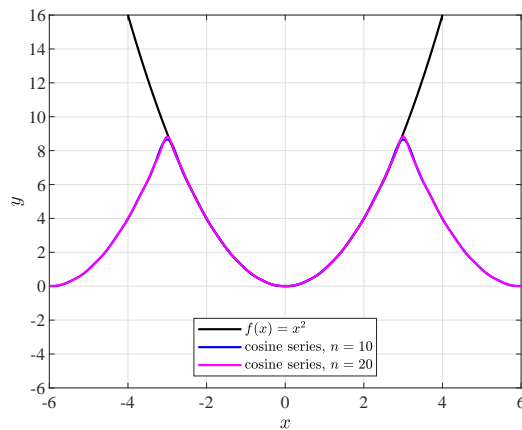
and

$$\begin{aligned} a_n &= \frac{2}{3} \int_0^3 x^2 \cos\left(\frac{n\pi x}{3}\right) dx, \\ &= \frac{2x^2 \sin\left(\frac{n\pi x}{3}\right)}{n\pi} + \frac{12x \cos\left(\frac{n\pi x}{3}\right)}{n^2\pi^2} - \frac{36 \sin\left(\frac{n\pi x}{3}\right)}{n^3\pi^3} \Bigg|_0^3, \\ &= \frac{36 \cos(n\pi)}{n^2\pi^2}, \\ &= \frac{36(-1)^n}{n^2\pi^2}. \end{aligned}$$

4. Below are graphs for $f(x)$ with $x \in [-6, 6]$ along with the Fourier cosine series with 3, 5, 10, and 20 terms.



$f(x)$ and Fourier cosine with $n = 3$ and 5



$f(x)$ and Fourier cosine with $n = 10$ and 20

Below is the MatLab program to generate the Fourier sine curves.

```

1  % Periodic Fourier sine series x^2, -6 < x < 6
2
3  NptsX=2000;           % number of x pts
4  x = linspace(-6,6,NptsX);
5  f = x.^2;
6
7  f1 = zeros(1,NptsX);
8  for n=1:3
9      b(n)=18*((2-n^2*pi^2)*(-1)^(n-2))/(n^3*pi^3); % Fourier sine coefficients
10     fn=b(n)*sin(n*pi*x/3); % Fourier function(n)
11     f1=f1+fn;
12 end
13 f2 = zeros(1,NptsX);
14 for n=1:5
15     b(n)=18*((2-n^2*pi^2)*(-1)^(n-2))/(n^3*pi^3); % Fourier sine coefficients
16     fn=b(n)*sin(n*pi*x/3); % Fourier function(n)

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17     f2=f2+fn;
18 end
19 f3 = zeros(1,NptsX);
20 for n=1:10
21     b(n)=18*((2-n^2*pi^2)*(-1)^n -2)/(n^3*pi^3); % Fourier sine coefficients
22     fn=b(n)*sin(n*pi*x/3); % Fourier function(n)
23     f3=f3+fn;
24 end
25
26 plot(x,f,'k-','LineWidth',1.5);
27 hold on
28 plot(x,f1,'b-','LineWidth',1.5);
29 plot(x,f2,'m-','LineWidth',1.5);
30 plot(x,f3,'r-','LineWidth',1.5);
31 grid;
32 h = legend('$f(x) = x^2$', 'sine series, $n = 3$', 'sine series, $n = 5$',...
33     'sine series, $n = 10$', 'Location','south');
34 set(h,'Interpreter','latex')
35 h.FontSize = 10;
36 xlim([-6,6]);
37 ylim([-16 16]);
38 xlabel('$x$', 'FontSize',12,'interpreter','latex');
39 ylabel('$y$', 'FontSize',12,'interpreter','latex');
40 set(gca,'FontSize',12); % Axis tick font size
41 print -depsc sinx2a.eps

```

Below is the MatLab program to generate the Fourier cosine curves.

```

1 % Periodic Fourier cosine series  $x^2$ ,  $-6 < x < 6$ 
2
3 NptsX=2000; % number of x pts
4 x = linspace(-6,6,NptsX);
5 f = x.^2;
6
7 a0 = 3*ones(1,NptsX);
8 f1 = zeros(1,NptsX);
9 for n=1:3
10     a(n)=36*(-1)^n/(n^2*pi^2); % Fourier cosine coefficients
11     fn=a(n)*cos(n*pi*x/3); % Fourier function(n)
12     f1=f1+fn;
13 end
14 f1 = a0+f1;
15 f2 = zeros(1,NptsX);
16 for n=1:5
17     a(n)=36*(-1)^n/(n^2*pi^2); % Fourier sine coefficients
18     fn=a(n)*cos(n*pi*x/3); % Fourier function(n)
19     f2=f2+fn;
20 end
21 f2 = a0+f2;
22
23 plot(x,f,'k-','LineWidth',1.5);
24 hold on
25 plot(x,f1,'b-','LineWidth',1.5);
26 plot(x,f2,'m-','LineWidth',1.5);
27 grid;
28 h = legend('$f(x) = x^2$', 'cosine series, $n = 3$', 'cosine series, $n = ...
29     '5$',...
30     'Location','south');

```

```
30 set(h, 'Interpreter', 'latex')
31 h.FontSize = 10;
32 xlim([-6, 6]);
33 ylim([-6 16]);
34 xlabel('$x$', 'FontSize', 12, 'interpreter', 'latex');
35 ylabel('$y$', 'FontSize', 12, 'interpreter', 'latex');
36 set(gca, 'FontSize', 12);           % Axis tick font size
37 print -depsc cosx2a.eps
```