1. (4pts) The initial value problem:

$$\frac{dy}{dt} = \frac{3+2t}{2y}, \qquad y(0) = -4,$$

is clearly separable. It follows that

$$\int 2y\,dy = \int (2t+3)\,dt \qquad \text{or} \qquad y^2 = t^2 + 3t + C.$$

Solving for y gives:

$$y(t) = \pm\sqrt{t^2+3t+C}, \qquad \text{so} \qquad y(0) = -4 = \pm\sqrt{C} \qquad \text{or} \qquad C = 16.$$

Taking the negative branch of the square root function from the IC yields:

$$y(t) = -\sqrt{t^2+3t+16}.$$

2. (6pts) a. For $H(t)$ being the temperature of the wine and $t = 0$ corresponding to 11 AM, Newton's Law of cooling and the data give the IVP:

$$\frac{dH}{dt} = -k(H-32) \qquad \text{or} \qquad \frac{dH}{dt} + kH = 32k, \qquad H(0) = 88,$$

which is a linear problem with integrating factor, $\mu(t) = e^{kt}$. It follows that

$$\frac{d}{dt}\left(e^{kt}H\right) = 32k\,e^{kt}, \qquad \text{so} \qquad e^{kt}H(t) = 32\,e^{kt} + C.$$

Thus, $H(t) = 32 + Ce^{-kt}$. The initial condition gives $88 = 32 + C$ or $C = 56$, so

$$H(t) = 32 + 56\,e^{-kt}.$$

From the information at $t = 30$, we have

$$H(30) = 72 = 32 + 56\,e^{-30k} \qquad \text{or} \qquad 30k = \ln\left(\frac{56}{40}\right) \qquad \text{or} \qquad k = \frac{1}{30}\ln\left(\frac{56}{40}\right) \approx 0.01122.$$

The wine should be served at 45°F, so

$$H(t_s) = 45 = 32 + 56\,e^{-0.01122t_s} \qquad \text{or} \qquad 0.01122t_s = \ln\left(\frac{56}{13}\right) \qquad \text{or} \qquad t_s \approx 130.21 \text{ min.}$$

It follows that the Riesling should be ready to drink at about 1:10 PM.

b. (6pts) The experimental study of the cooling gives the separable model:

$$\frac{dH_b}{dt} = -k_b(H_b - 32)^{3/4}.$$

Separating variables yields:

$$\int (H_b - 32)^{-3/4} dH = -k_b \int dt = -k_b t + C, \qquad \text{so} \qquad 4(H_b - 32)^{1/4} = -k_b t + C.$$

With the IC, it follows that

$$H_b(t) = 32 + \left(\frac{C - k_b t}{4}\right)^4 \qquad \text{or} \qquad 88 - 32 = 56 = \left(\frac{C}{4}\right)^4 \qquad \text{or} \qquad \frac{C}{4} = 56^{1/4} \approx 2.7356.$$

Thus, the solution is given by

$$H_b(t) = 32 + \left(56^{1/4} - \frac{k_b t}{4}\right)^4.$$

From the condition at 11:30, we have that $H_b(30) = 72 = 32 + \left(56^{1/4} - \frac{30k_b}{4}\right)^4$ or $56^{1/4} - 40^{1/4} = \frac{30k_b}{4}$ or

$$k_b = \frac{4\left(56^{1/4} - 40^{1/4}\right)}{30} \approx 0.02943.$$

The wine should be served at 45°F, so $H_b(t_s) = 45 = 32 + \left(56^{1/4} - \frac{0.02943\, t_s}{4}\right)^4$ or $\frac{0.02943\, t_s}{4} = 56^{1/4} - 13^{1/4}$ or

$$t_s \approx 113.74 \text{ min.}$$

It follows that the Riesling should be ready to drink at about 12:54 PM.