1. This problem has you use nonlinear computer solvers to find points of intersection and appreciate the differences in growth of polynomials, exponentials, logarithms, and fractional powers. Be sure to tell how you found your points of intersection.

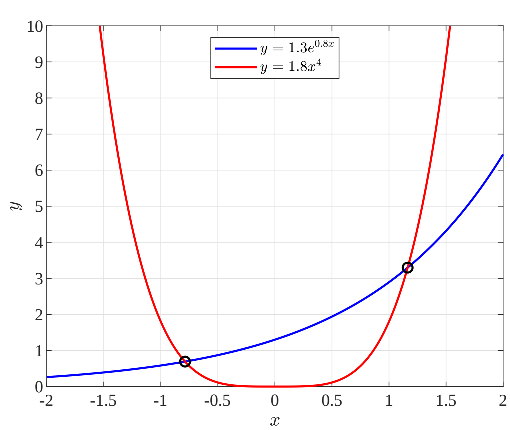
a. (3pts) The functions:

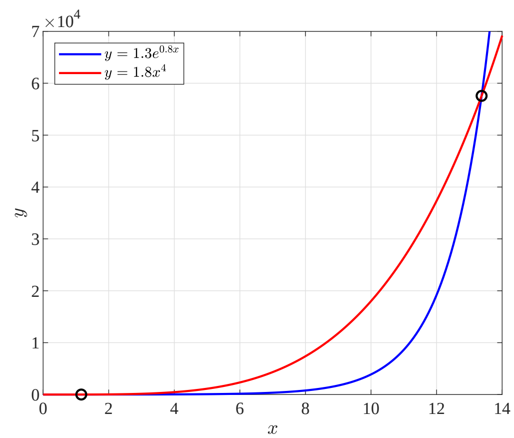
$$f(x) = 1.3e^{0.8x} \quad \text{and} \quad g(x) = 1.8x^4,$$

are highly nonlinear so can only be solved numerically. The three points of intersection for these functions are found using *fsolve* in either Maple or MatLab and are given by:

$$(-0.787524, 0.692356), \qquad (1.16337, 3.29715), \qquad (13.3729, 57568.0).$$

Graphs of points of intersection are shown below, requiring two scales to visualize.





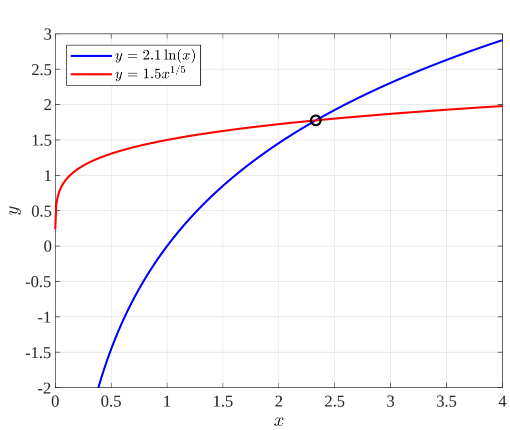
b. (2pts) The functions:

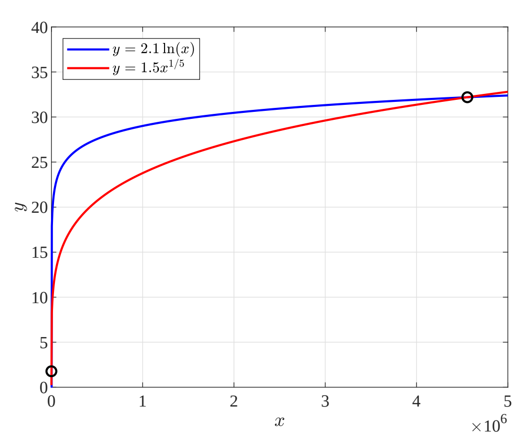
$$h(x) = 2.1\ln(x) \quad \text{and} \quad k(x) = 1.5x^{1/5},$$

are also highly nonlinear so can only be solved numerically. The two points of intersection for these functions are found using *fsolve* in either Maple or MatLab and are given by:

$$(2.33022, 1.77652) \qquad \text{and} \qquad (4556690, 32.1974).$$

Graphs of points of intersection are shown below, requiring two scales to visualize.





2. (3pts) The IVP given by:

$$\frac{dy}{dt} = t\,\cos(t^2), \qquad y(0) = 3,$$

has an ODE with only a time dependent function on the rhs, so is solved by integrating (by substitution). It follows that

$$y(t) = \int t\,\cos(t^2)dt = \frac{1}{2}\sin(t^2) + C.$$

The IC $y(0) = 3$ give $C = 3$, so

$$y(t) = \frac{1}{2}\sin(t^2) + 3.$$

3. (4pts) The IVP given by:

$$\frac{dy}{dt} - 2ty = 12\,t, \qquad y(0) = 5,$$

is a linear ODE with an integrating factor,

$$\mu(t) = exp\left(-\int 2t\,dt\right) = e^{-t^2}.$$

Multiplying the ODE by the integrating factor gives:

$$\frac{d}{dt}\left(e^{-t^2}y(t)\right) = 12t\,e^{-t^2},$$

which upon integration gives

$$e^{-t^2}y(t) = \int 12t\,e^{-t^2}dt = -6\,e^{-t^2} + C.$$

The IC $y(0) = 5$ gives $5 = -6 + C$ or $C = 11$, so

$$y(t) = 11\,e^{t^2} - 6.$$

4. (4pts) The linear ODE given by:

$$t\frac{dy}{dt} - 2y = 4\,t^3\sin(4t) \qquad \text{is} \qquad \frac{dy}{dt} - \frac{2}{t}y = 4\,t^2\sin(4t).$$

Thus, the integrating factor,

$$\mu(t) = exp\left(-\int \frac{2}{t}\,dt\right) = \frac{1}{t^2}.$$

Multiplying the ODE by the integrating factor gives:

$$\frac{d}{dt}\left(\frac{y(t)}{t^2}\right) = 4\,\sin(4t),$$

which upon integration gives

$$\frac{y(t)}{t^2} = \int 4\,\sin(4t)dt = -\cos(4t) + C.$$

The IC $y(1) = 2$ gives $2 = -\cos(4) + C$ or $C = 2 + \cos(4)$, so

$$y(t) = -t^2\cos(4t) + t^2(2 + \cos(4)).$$