

This Lecture Activity is designed to have you actively work with the lecture notes presented in class and available on my website. This activity is meant to keep you engaged and current with the class, so there is a fairly rapid turn around (due by **Thur. Feb 17 by noon**). There are 3 problems that require written answers, which are entered into **Gradescope**.

**Note:** For full credit you must show intermediate steps in your calculations.

1. (6pts) a. An initially clean lake ( $c(0) = 0$ ) maintains a constant volume of  $V = 400,000 \text{ m}^3$  of water. Above this lake is a stream feeding in from agricultural fields that have been sprayed with a new pesticide. This stream has a flow rate of  $f = 800 \text{ m}^3/\text{day}$ . With several measurements it is found that the pesticide concentration in the stream satisfies  $p(t) = 20 e^{-0.0005t} \mu\text{g}/\text{m}^3$ . Assume that this is a well-mixed lake with a stream flowing out at the same rate of  $f$  (with the pesticide in the outflowing stream equal to the concentration in the lake). Write a differential equation describing the concentration of pesticide in the lake ( $c(t)$ ) and solve this differential equation.

b. Create a graph of the solution, showing the concentration of the pesticide in the lake as a function of time. When is the concentration of the pesticide at its maximum concentration and what is that concentration? (Slides Linear 24–35)

2. (5pts) a. The blue shark *Prionace glauca* grows according to von Bertalanffy's growth equation:

$$\frac{dL}{dt} = b(240 - L),$$

where  $L$  is the length in cm. Like many sharks, it is viviparous giving birth to a developed shark pup, which we assume satisfies  $L(0) = 24 \text{ cm}$ . Solve this initial value problem.

b. Assume that a 10 yr old blue shark satisfies  $L(10) = 196 \text{ cm}^1$ . Find the value of  $b$ . Graph the solution for  $t \in [0, 20]$  and note any horizontal asymptotes. (Slides Linear 41–48)

3. (5pts) Consider the data on a collection of various sized mammals given below.

Animal	Weight	Metabolism	Animal	Weight	Metabolism
Mouse	0.021	3.6	Dog	23.6	872
Rat	0.282	28.1	Chimpanzee	38	1090
Guinea pig	0.41	35.1	Sheep	46.8	1330
Rabbit	3.57	154	Woman	57.2	1368
Cat	3	152	Heifer	482	7754

The weight,  $w$ , is in kg and the metabolism,  $M$ , is in kcal. Use an allometric or power law model to find Kleiber's Law:

$$M = Aw^r,$$

which relates the weight of an animal to its metabolism. Find the best linear fit to the logarithms of the data to obtain the best  $A$  and  $r$ . Create a graph with both the data and best linear fit, using

<sup>1</sup>Y. Fujinami, Y. Semba, S. Tanaka (2019), Age determination and growth of the blue shark (*Prionace glauca*) in the western North Pacific Ocean, *Fishery Bull.*, **117**, pp 107-120

the logarithmic scales for  $w$  and  $M$ . Create a second graph of the data and the allometric model using normal scales for  $w$  and  $M$ . Give a short discussion for why you obtain the value of  $r$  that you find, using your knowledge of how animals use their energy based on their size. (Slides Linear 49–52)