In class we studied the growth of *Escherichia coli* with the discrete Malthusian growth,

$$P_{n+1} = (1+r)P_n, \qquad \text{with IC} \quad P_0, \tag{1}$$

and continuous Malthusian ODE models,

$$\frac{dP}{dt} = kP, \qquad \text{with IC} \quad P(0) = P_0. \tag{2}$$

Early stage agrarian societies often grow according to Malthusian growth. This computer activity uses early U. S. census data to fit the two models above.

Below is a table of the U. S. census data, giving the census date and population in millions.

Year	Population	Year	Population	Year	Population
1790	3.929	1820	9.638	1850	23.192
1800	5.308	1830	12.866	1860	31.433
1810	7.240	1840	17.069	1870	39.818

Table 1: U. S. census data from 1790 to 1870 with the population in millions.

a. Begin with the continuous Malthusian ODE model, (2), and find the general solution to this model, including the parameters, k and P_0, where 1790 corresponds to $t = 0$ in years. Use the data in Table 1 to find the best fitting parameters in the *nonlinear least squares* sense, using Excel's Solver. Write your best fitting model with the appropriate parameters and give the least sum of square errors (LSSE). In a short paragraph describe the key steps for Excel you performed to find this best fitting model and include a sentence stating how well this model fits the data. Present an Excel graph of the data and the model, including a title and appropriately labeled axes.

b. Repeat the steps in Part a with the continuous Malthusian ODE model, (2), using MatLab's fminsearch. Again write your best fitting model from MatLab with the appropriate parameters and give the least sum of square errors (LSSE). In a short paragraph describe the key steps for MatLab you performed to find this best fitting model and include a sentence stating how well this model fits the data. Present a MatLab graph of the data and the model, including a title, appropriately labeled axes, and a legend.

c. Now consider the discrete Malthusian model, (1). Simulate this model in Excel, including the named parameters, r and P_0, where n is in decades after 1790. (Do NOT use the solution to the discrete Malthusian growth model.) Use the data in Table 1 to adjust the parameters to find the simulation that fits this model best in the *nonlinear least squares* sense, using Excel's Solver. Write your best fitting model with the appropriate parameters and give the least sum of square errors (LSSE). In a short paragraph describe the key steps you performed in Excel to find this best fitting model and include a sentence stating how well this model fits the data. Present an Excel graph of the data and the model, including a title and appropriately labeled axes.

d. Repeat the steps in Part c with the discrete Malthusian model, (2), using MatLab's fminsearch. Again write your best fitting model from MatLab with the appropriate parameters and give the least sum of square errors (LSSE). In a short paragraph describe the key steps in MatLab you performed to find this best fitting model and include a sentence stating how well this model fits the data. Present a MatLab graph of the data and the model, including a title, appropriately labeled axes, and a legend.

e. Write a brief summary of the pros and cons of using Excel or MatLab for these models. Give a couple of positive and negative reasons for each of these programs for solving the problems you worked above. Malthusian growth is an exponential growth, and the discrete model, (2), has a closed form solution (see lecture notes). It follows these models are equivalent. Give an algebraic expression relating the parameters k to r ($k = f(r)$ for some f). Are there differences between the results using Excel and the ones from MatLab? Explain your answer and give a possible reason.