Spring 2020 Math 531 Name ______________________

Lecture Notes 8: Fourier Transforms

Note: For full credit you must show intermediate steps in your calculations.

1. (3pts) Find the Fourier transform of the following function:

$$f(x) = \begin{cases} e^{-ax}, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where $a > 0$. Show your integrations. This result gives you one *transform pair for a Fourier transform* table.
(Slide 15-17)

2. (4pts) The Fourier sine transform is defined by:

$$F(\omega) = \frac{2}{\pi}\int_0^\infty f(x)\sin(\omega x)dx,$$

while its inverse transform is given by:

$$f(x) = \int_0^\infty F(\omega)\sin(\omega x)d\omega.$$

Consider $F(\omega) = e^{-\beta\omega}$, $\beta > 0$ ($\omega \geq 0$). Find the inverse Fourier sine transform by evaluating:

$$f(x) = \int_0^\infty e^{-\beta\omega}\sin(\omega x)d\omega.$$

Show your integration methods in solving this problem (not Maple). This result gives you one *transform pair for a Fourier sine transform* table.
(Slide 15-17)

3. (3pts) Suppose that

$$f(x) = \int_{-\infty}^{\infty} F(\omega)e^{-i\omega x}\,d\omega, \qquad \text{and} \qquad F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{i\omega x}\,dx.$$

Find the Fourier transform of $xf(x)$. (Hint: Try differentiating $F(\omega)$ with respect to ω.)
(Slides 15-17).

4. (6pts) The Gamma function, $\Gamma(x)$ is defined:

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt.$$

a. Show the following:

$$\Gamma(x+1) = x\Gamma(x), \qquad \text{and} \qquad \Gamma(n+1) = n!$$

b. Evaluate the following:

$$\Gamma\left(\tfrac{1}{2}\right), \qquad \text{and} \qquad \Gamma\left(\tfrac{3}{2}\right).$$

Hint: The first of these can be solved with the substitution, $t = u^2$.