Spring 2020 Math 531 Name ______________________

Lecture Notes 7: Nonhomogeneous PDEs

Note: For full credit you must show intermediate steps in your calculations.

Consider the nonhomogeneous PDE in a circular domain:

$$\frac{\partial u}{\partial t} = k\nabla^2 u + f(r,t) = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + f(r,t), \qquad 0 < r < a, \quad t > 0, \tag{1}$$

with homogeneous BC and IC:

$$u(a,t) = 0 \qquad \text{and} \qquad u(r,0) = 0.$$

1. (7pts) Consider the PDE (1) with its homogeneous BCs. Find the eigenfunctions $\phi_n(r)$ and write an eigenfunction expansion:

$$u(r,t) = \sum_{n=1}^{\infty} a_n(t)\phi_n(r).$$

Use principles of orthogonality to find a linear nonhomogeneous ODE for the coefficients $a_n(t)$. From the initial condition find $a_n(0)$, then write the solution for $a_n(t)$, which will be in integral form (variation of parameters formula). (Slides 10-14)

2. (4pts) Continuing with this nonhomogeneous PDE (1) give the appropriate *Green's function*, $G(r,t;r_0,t_0)$. With this Green's function, write the solution to (1), $u(r,t)$ (much like Slide 28). (Slides 24-28).

3. (4pts) How would your answers to this problem change if the boundary condition changed from the fixed above to insulated (Dirichlet above to Neumann), that is:

$$\frac{\partial u}{\partial r}(a,t) = 0.$$

Comment on the solvability of this problem or complications in the physical solution.