Spring 2020 Math 531 Name ______________________________

Lecture Notes 6: Nonhomogeneous PDEs

Note: For full credit you must show intermediate steps in your calculations.

Consider the nonhomogeneous PDE:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + q\cos(7x), \tag{1}$$

with nonhomogeneous BC's and IC:

$$\frac{\partial u}{\partial x}(0,t) = A, \qquad u\left(\tfrac{\pi}{2},t\right) = B, \qquad \text{and} \qquad u(x,0) = f(x).$$

1. (5pts) Consider the PDE (1) with its nonhomogeneous BCs. Find the equilibrium solution $u_E(x)$ to (1) satisfying $u_E'(0) = A$ and $u_E\left(\frac{\pi}{2}\right) = B$. (Slides 3-4)

2. (4pts) Returning to the PDE (1) with its nonhomogeneous BCs, we find a simple linear *reference temperature distribution*, $r(x)$, so that if

$$u(x,t) = v(x,t) + r(x),$$

then $v(x,t)$ satisfies (1) with related homogeneous BCs and a slightly different initial condition. Give the appropriate *linear* $r(x)$ and show the complete nonhomogeneous PDE in $v(x,t)$ with its BCs and ICs. (Slides 8-10).

3. (7pts) Take the nonhomogeneous PDE in $v(x,t)$ with its homogeneous BCs and revised ICs and use the *method of eigenfunction expansion* to find the solution $v(x,t)$, which in turn solves our original problem (1) for $u(x,t)$. You must find the appropriate eigenfunctions, $\phi_n(x)$ for this PDE in $v(x,t)$, then let

$$v(x,t) = \sum_{n=1}^{\infty} a_n(t)\phi_n(x),$$

and solve for the *time-varying coefficients*, $a_n(t)$. (Slide 11-17)