Spring 2020 Math 531 Name ______________________

Lecture Notes 3: Laplace's Equation on a Cylinder

Note: For full credit you must show intermediate steps in your calculations.

1. (8pts) Consider Laplace's equation for a cylinder:

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

with the boundary conditions:

$$u(r,\theta,0) = \alpha(r)\cos(2\theta), \quad u_z(r,\theta,H) = 0, \quad \text{and} \quad u(a,\theta,z) = 0.$$

Find the solution for this problem, noting differences from the problem in the notes (Slides 6-12) and writing the answer in the simplest form (showing only nonzero Fourier coefficients).

2. (8pts) Consider Laplace's equation for a cylinder with no θ dependence:

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} = 0.$$

with the boundary conditions:

$$u_z(r,0) = 0, \quad u_z(r,H) = 0, \quad \text{and} \quad u(a,z) = g(z).$$

Find the solution for this problem, noting differences from the problem in the notes (Slides 14-17) and writing the answer in the simplest form (showing only nonzero Fourier coefficients).