Spring 2020 Math 531 Name ______________________

**Lecture Notes 2: Vibrating Circular Membrane**

**Note:** For full credit you must show intermediate steps in your calculations.

1. (2pts) On Slides 9, we see the statement that the substitution $z = \sqrt{\lambda}r$ transforms the singular Sturm-Liouville problem at the top of the slide into Bessel's equation for $\phi(z)$. Show details of this transformation.

2. (5pts) The Slides 10-11 of the notes show steps for obtaining the power series for $J_0(z)$ using methods very similar to the power series methods of Math 337. Repeat these steps for the case $m = 1$. Repeat finding the *indicial equation* for this particular case. Use $r = 1$, then find the *recurrence relation* and write out a few terms and see if you see a pattern to obtain the power series for $J_1(z)$.

3. (3pts) Create a graph of $J_1(z)$ and $Y_1(z)$ (using any software you like) that is similar to the graph on Slide 13.

4. (6pts) Consider the heat equation on a circular disk, where the region is circularly symmetric, $u = u(r,t)$:

$$\frac{\partial u}{\partial t} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right),$$

with BCs:

$$u(a,t) = 0, \qquad \text{and} \quad |u(0,t)| < \infty,$$

and IC:

$$u(r,0) = \alpha(r).$$

Repeat the steps of Slides 30-33 modified for this equation to obtain the solution of the circularly symmetric heat equation.