HW 1

1.2.9. Consider a thin one-dimensional rod without sources of thermal energy whose lateral surface area is not insulated.

(a) Assume that the heat energy flowing out of the lateral sides per unit surface area per unit time is $w(x,t)$. Derive the partial differential equation for the temperature $u(x,t)$.

(b) Assume that $w(x,t)$ is proportional to the temperature difference between the rod $u(x,t)$ and a known outside temperature $\gamma(x,t)$. Derive that

$$c\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K_0 \frac{\partial u}{\partial x}\right) - \frac{P}{A}[u(x,t) - \gamma(x,t)]h(x), \tag{1.2.15}$$

where $h(x)$ is a positive x-dependent proportionality, P is the lateral perimeter, and A is the cross-sectional area.

(c) Compare (1.2.15) to the equation for a one-dimensional rod whose lateral surfaces are insulated, but with heat sources.

(d) Specialize (1.2.15) to a rod of circular cross section with constant thermal properties and 0° outside temperature.

*(e) Consider the assumptions in part (d). Suppose that the temperature in the rod is uniform [i.e., $u(x,t) = u(t)$]. Determine $u(t)$ if initially $u(0) = u_0$.

1.4.12. Suppose the concentration $u(x,t)$ of a chemical satisfies Fick's law (1.2.13), and the initial concentration is given $u(x,0) = f(x)$. Consider a region $0 < x < L$ in which the flow is specified at both ends $-k\frac{\partial u}{\partial x}(0,t) = \alpha$ and $-k\frac{\partial u}{\partial x}(L,t) = \beta$. Assume α and β are constants.

(a) Express the conservation law for the entire region.

(b) Determine the total amount of chemical in the region as a function of time (using the initial condition).

(c) Under what conditions is there an equilibrium chemical concentration and what is it?