Spring 2020 Math 337 - Lecture 8 Name _____

Note: For full credit you must show intermediate steps in your calculations. Your work must be your own. Copying or sharing solutions with others may subject you to disciplinary action based on the appropriate sections of the San Diego State University Policies.

1. (4pts) The Fourier sine transform is defined by:

$$F(\omega) = \frac{2}{\pi}\int_0^\infty f(x)\sin(\omega x)dx,$$

while its inverse transform is given by:

$$f(x) = \int_0^\infty F(\omega)\sin(\omega x)d\omega.$$

Consider $F(\omega) = e^{-\beta\omega}$, $\beta > 0$ ($\omega \geq 0$). Find the inverse Fourier sine transform by evaluating:

$$f(x) = \int_0^\infty e^{-\beta\omega}\sin(\omega x)d\omega.$$

Show your integration methods (integration by parts) in solving this problem. This result gives you one *transform pair for a Fourier sine transform* table. (Slide 4)

2. (4pts) Use the definition of the Laplace transform to find

$$\mathcal{L}\{\cosh(\beta t)\}, \qquad s > \beta,$$

i.e., form the integrals in the definition and solve them. Use the definition of $\cosh(\beta t)$ in terms of the appropriate sum of exponentials to work your integrals. Write your answer with one common denominator. (Hint: Your answer should have some similarity to the Laplace Table entry for $\cos(\beta t)$.) (Slide 7-8)

3. (4pts) Use the result in Question 2 to solve the initial value problem with *Laplace transforms*:

$$y'' - 9y = 0, \qquad y(0) = 6, \quad y'(0) = 0.$$

Thus, your answer should include the cosh function. (Slide 23-25)

4. (4pts) Solve the following initial value problem with *Laplace transforms*:

$$y'' + 2y' + y = 12te^{-t}, \qquad y(0) = 3, \quad y'(0) = -2.$$

Use the Laplace table (https://jmahaffy.sdsu.edu/courses/s20/math337/hwprob/Reviews/Laplace Table.pdf) provided, stating clearly which elements of the Table or Theorems in this section that you are using for your solution. (Slide 21-25)