Spring 2020 Math 337 Name ______________________

Note: For full credit you must show intermediate steps in your calculations. Your work must be your own. Copying or sharing solutions with others may subject you to disciplinary action based on the appropriate sections of the San Diego State University Policies.

1. (5pts) Consider the 3^{rd} order linear homogeneous ODE given by:

$$t^2y''' - ty'' + 2y' = 0.$$

Use similar techniques for solving the *Cauchy-Euler problem* to solve this problem. Find **3** linearly independent solutions to this problem. How would one establish that these are **3** linearly independent solutions. (Slides 3-10)

2. (5pts) **Reduction of Order** (Jean D'Alembert (1717-1783)): If $y_1(x)$ is known for the linear ODE:

$$y'' + p(x)y' + q(x)y = 0.$$

Then one attempts a solution of the form $y(x) = v(x)y_1(x)$. Provided $y_1(x) \neq 0$, show that

$$\frac{dv}{dx} = \frac{1}{[y_1(x)]^2}e^{-\int^x p(s)ds}.$$

Solve for $v(x)$ to obtain the 2^{nd} linearly independent solution, $y_2(x)$. (This will just be an integral expression, not easily simplified.)

3. (6pts) a. Consider the following ODE:

$$xy'' + (1-2x)y' + (x-1)y = 0. \tag{1}$$

Show that $y_1(x) = e^x$ is a solution to this differential equation.

b. In Part a, $y_1(x) = e^x$ was found as one solution to (1). Use the **Reduction of Order** method to find $y_2(x)$ for (1). Use the Wronskian to show this is a *fundamental set of solutions*.