Spring 2020 Math 337 Name _____

Note: For full credit you must show intermediate steps in your calculations. Your work must be your own. Copying or sharing solutions with others may subject you to disciplinary action based on the appropriate sections of the San Diego State University Policies.

1. (5pts) Consider the 2^{nd} order linear homogeneous ODE given by:

$$y'' + 4y' + 4y = 24te^{-2t} + 40\cos(2t).$$

Use the *Method of Undetermined Coefficients* to solve this problem. (Slides 22-24)

2. (4pts) For the following nonhomogeneous differential equation give the form of the particular solution that you would guess in using the **method of undetermined coefficients**. Include your solution to the homogeneous problem. (**DO NOT** solve for the undetermined coefficients.)

$$y'' - 2\,y' + y = 5t\,e^t \sin(2t) + 20t^2 e^t.$$

(Slide 24)

3. (7pts) A crude tuning device can be created by an LRC-circuit forced by an external signal. An LRC-circuit is equipped with a variable capacitor, which can be dialed to different values to obtain the maximum response from an incoming radio signal. If $I(t)$ is the current in the tuning device, which contains an inductor, L, a resistor, R, and a tunable capacitor, C, and receives an external signal, $V_0\omega\cos(\omega t)$, the ODE describing this system satisfies:

$$L\ddot{I} + R\dot{I} + \tfrac{1}{C}I = V_0\omega\cos(\omega t), \tag{1}$$

where V_0 is the strength of the signal and $\omega = 2\pi f$ and f is the frequency of the signal.

a. Suppose the inductor is $L = 30$ mH, the resistor is $R = 10\ \Omega$, the maximum signal is $V_0 = 50$, and the frequency $f = 60$ Hz. Solve Eqn. (1) for any C. Give the solution for $t \to \infty$. Find the amplitude of this oscillatory solution.

b. Find the value of C that gives the optimal response of this circuit to the external signal above, C_{max}, and determine the amplitude of that response. With this value, C_{max}, of tuning, what is the magnitude of the response to a $f = 50$ Hz signal. (Slides 30-32)