Spring 2020 Math 337 Name ____________________

Note: For full credit you must show intermediate steps in your calculations. Your work must be your own. Copying or sharing solutions with others may subject you to disciplinary action based on the appropriate sections of the San Diego State University Policies.

1. (3pts) Consider the 2^{nd} order linear homogeneous ODE given by:

$$y'' + 6y' + 13y = 0.$$

Find two linearly independent solutions, y_1 and y_2, for this ODE and write the general solution to this problem. Show these solutions form a *Fundamental set of solutions* by computing the Wronskian, $W[y_1, y_2](t)$ and showing it is nonzero for all t. (Slides 10-14)

2. (5pts) Consider the 2^{nd} order linear homogeneous ODE given by:

$$y'' - y' - 2y = 54t\, e^{2t} - 20t.$$

Find the general solution to this problem, using the *Method of Undetermined Coefficients*. You must show your steps for finding the coefficients of the *particular solution.* (Slides 18-24)

3. a. (3pts) An important 2^{nd} order nonlinear homogeneous ODE shown on Slide 7 describes the motion of a pendulum and satisfies:

$$\theta'' + 0.2\theta' + 4.01\sin(\theta) = 0,$$

where $\theta(t)$ is the angle of the pendulum from the downward vertical. Transform this 2^{nd} order nonlinear ODE into a system of 1^{st} order ODEs by letting $x_1(t) = \theta(t)$ and $x_2(t) = \dot{x}_1(t) = \theta'(t)$. Find all equilibria by letting $\dot{x}_1 = \dot{x}_2 = 0$. (Slides 6-7)

b. (5pts) Take the nonlinear system of 1^{st} order ODEs found in Part a and determine the Jacobian matrix, $J(x_1, x_2)$, for this system (as we did in the previous section 2D Linear Systems Applications). One equilibrium is $[x_{1e}, x_{2e}]^T = [0, 0]^T$, so compute $J(0, 0)$. Find the eigenvalues for $J(0, 0)$ and use this information to determine the qualitative behavior (*e.g.*, stable node, center, etc.) near this equilibrium, as we did in the previous section. Another equilibrium is $[x_{1e}, x_{2e}]^T = [\pi, 0]^T$, so compute $J(\pi, 0)$. Find the eigenvalues for $J(\pi, 0)$ and use this information to determine the qualitative behavior near this equilibrium. (For example, Slides 46-49 of 2D Linear Systems Applications)