Spring 2020 Math 337 Name ______________________________

Note: For full credit you must show intermediate steps in your calculations.

1. (8pts) The lecture notes examined the negative feedback of glucose and insulin. A classic enzymatic negative feedback model satisfies the system:

$$\begin{aligned}\dot{x}_1 &= \frac{3}{1+0.2x_2} - 0.5x_1,\\ \dot{x}_2 &= 5x_1 - x_2,\end{aligned}$$

where x_1 is an enzyme and x_2 is the endproduct. Find the positive equilibrium for this model ($x_1 > 0$ and $x_2 > 0$). Compute the Jacobian Matrix for this system. Evaluate the Jacobian matrix at the equilibrium. Determine the eigenvalues for this model and determine the qualitative behavior of this model near the equilibrium. Sketch a phase portrait for this model for non-negative x_1 and x_2 ($x_1 \geq 0$ and $x_2 \geq 0$). (Slides 25-31)

2. (8pts) A very popular ecological model is the predator-prey model (Lotka-Volterra). Consider the system of ODEs:

$$\begin{aligned}\dot{x}_1 &= 0.1x_1 - 0.05x_1x_2,\\ \dot{x}_2 &= 0.001x_1x_2 - 0.04x_2.\end{aligned}$$

Associate each variable with the prey or the predator and explain briefly your reasoning. Find all equilibria for this model. Compute the Jacobian Matrix for this system. Evaluate the Jacobian matrix at all of the equilibria. Determine the eigenvalues at each of the equilibria for this model and determine the qualitative behavior of this model near these equilibria. Sketch a phase portrait for this model for non-negative x_1 and x_2 ($x_1 \geq 0$ and $x_2 \geq 0$). (Slides 45-54)