Spring 2020 Math 337 Name ______________________

Note: For full credit you must show intermediate steps in your calculations.

1. (4pts) Consider the initial value problem (IVP):

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \qquad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

Find the general solution to this problem, create a phase portrait, and solve the initial value problem. Describe the *qualitative behavior* shown in the phase portrait. (Slides 19-22)
Note: *Qualitative behavior* is *e.g., stable node, unstable spiral, saddle point, center, etc.*

2. (4pts) Consider the differential equation:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Find the general solution to this problem and create a phase portrait. Describe the qualitative behavior shown in the phase portrait. (Slides 44-49)

3. (4pts) Consider the differential equation with the parameter α:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \alpha & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Find the general solutions and create phase portraits for the values $\alpha = -6$ and $\alpha = 3$. Describe the qualitative behavior shown in the phase portraits. (Slides 50-54)

4. (4pts) Consider the differential equations $\dot{\mathbf{x}} = J_i\mathbf{x}$, where J_i is each of the following matrices:

$$J_1 = \begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}, \qquad J_2 = \begin{pmatrix} 5 & 3 \\ -2 & 2 \end{pmatrix}, \qquad J_3 = \begin{pmatrix} 1 & -3 \\ 2 & -5 \end{pmatrix}, \qquad J_4 = \begin{pmatrix} 3 & -2 \\ 6 & -3 \end{pmatrix},$$

Use the diagram on Slide 55 to classify the qualitative behavior for these differential equations ($J_i, i = 1, 2, 3, 4$) without solving the equations.