$\qquad$

Note: For full credit you must show intermediate steps in your calculations. Your work must be your own. Copying or sharing solutions with others may subject you to disciplinary action based on the appropriate sections of the San Diego State University Policies.

1. (5pts) Consider the Taylor series given by the following:

$$
f(x)=\sum_{n=1}^{\infty} \frac{(-1)^{n}(x-1)^{2 n-1}}{3 n-1}
$$

Find all values of $x$ where this series converges absolutely, diverges, or converges conditionally. Give the series test that shows the convergence or divergence. What is the radius of convergence for this series about $x=1$ ?
(Slides 9-14)
2. (11pts) Solve the following ODE with a power series method:

$$
\left(4-x^{2}\right) y^{\prime \prime}-x y^{\prime}+16 y=0 .
$$

Assume a power series solution of the form

$$
y(x)=\sum_{n=0}^{\infty} a_{n} x^{n} .
$$

Clearly state the recurrence relation. Determine all the coefficients $a_{n}$ for $n=2, \ldots, 10$ in terms of the two arbitrary constants, $a_{0}$ and $a_{1}$. Find the two linearly independent solutions, $y_{1}$ and $y_{2}$, up to and including terms of $x^{10}$. (You are not expected to find a closed form solution of any infinite series.) Determine all values of $x$ where your solutions $y_{1}$ and $y_{2}$ converge absolutely.
(Slide 15-24)

