Spring 2020 Math 337 - Lecture 11 Name ______________________________

Note: For full credit you must show intermediate steps in your calculations. Your work must be your own. Copying or sharing solutions with others may subject you to disciplinary action based on the appropriate sections of the San Diego State University Policies.

1. (4pts) Consider the periodic function $f(t)$ defined as follows:

$$f(t) = \begin{cases} t, & 0 \le t < 2, \\ 4 - t, & 2 \le t < 4, \end{cases} \qquad \text{with} \qquad f(t+4) = f(t).$$

Sketch a graph of this function for $t \in [0, 12]$. Write this function as a *window function*, $f_4(t)$, (Slide 22) using *step functions*. Use our Theorem (Slide 23) to obtain the Laplace Transform, $\mathcal{L}[f(t)] = F(s)$. This expression simplifies by dividing out a common factor in the numerator and denominator. Follow the example in the Lecture Slides to express the resulting rational expression in terms of a *geometric series*.
(Slides 23-27 and Laplace Table)

2. (8pts) Solve the following initial value problem with *Laplace transforms*:

$$y' + 4y = f(t), \qquad y(0) = 2,$$

where $f(t)$ is the periodic function given in the previous problem above. Show all the steps needed to find $\mathcal{L}[y(t)] = Y(s)$, then show the necessary *partial fractions decomposition (PFD)* required to make your elements appear in the *Laplace table*. Finally, invert $Y(s)$ to find your solution.
(Slide 23-27)

3. (4pts) Solve the following initial value problem with *Laplace transforms*:

$$y'' + 4y' + 5y = \frac{2t}{\pi}\left(\delta(t-\pi) - \delta(t-2\pi)\right), \qquad y(0) = 0, \quad y'(0) = 2.$$

Use the *Laplace table* to find your solution. Use the computer to create a graph of your solution for $t \in [0, 15]$. What is the limiting solution for large t?
(Slide 33-35)