March 3, 2017 Math 124a Name Key

1. (8pts) a. A small rodent is racing to escape a feline predator and must clear a small wall. The equation for the height of the rodent jumping to clear the wall is given by:

$$h(t) = V\,t - 16\,t^2,$$

where h is the height in ft, t is in sec, and V is the velocity of the jumping rodent. Suppose that the rodent jumps with a velocity of $V = 10$ ft/sec. Find the average velocity of the rodent between $t = 0.1$ and $t = 0.15$ sec. Also, determine the average velocity between $t = 0.1$ and $t = 0.11$ sec.

$$\frac{h(0.15)-h(0.1)}{0.05} = \frac{1.14-0.84}{0.05}$$

$$\frac{h(0.11)-h(0.1)}{0.01} = \frac{0.9064-0.84}{0.01}$$

For $t \in [0.1, 0.15]$, $v_{ave} =$ 6.0 ft/sec For $t \in [0.1, 0.11]$, $v_{ave} =$ 6.64 ft/sec

b. Given $V = 10$, find an expression for the velocity at any time, $v(t)$, then compute $v(0.1)$. Also, determine how high this rodent jumps and when this maximum height occurs (t_{max}).

$v(t) = V - 32t$ $\quad t_{max} = \frac{10}{32}$

$v(t) =$ $10 - 32t$ ft/sec $\quad v(0.1) =$ 6.8 ft/sec

$t_{max} =$ 0.3125 sec $\quad h(t_{max}) =$ 1.5625 ft

c. Suppose that the wall is 1.2 ft in height. Find the minimum velocity V that the rodent needs to clear this wall. Also, determine how long the rodent is in the air before it touches the ground ($h = 0$) on the other side., t_{out}.

$v(t) = V - 32t \quad t_{max} = \frac{V}{32} \qquad t_{out} = 2t_{max} = \frac{8.7636}{16}$

$h(\frac{V}{32}) = V(\frac{V}{32}) - 16(\frac{V}{32})^2 = \frac{V^2}{64} = 1.2 \Rightarrow V = 8\sqrt{1.2}$

$V =$ 8.7636 ft/sec $\quad t_{out} =$ 0.54772 sec

2. (3 pts) Consider the function:

$3x^{-4}$

$$f(x) = 8x^{\frac{7}{4}} - \frac{3}{x^4} + 16.$$

Find the derivative of this function (any means).

$f'(x) =$ $14x^{\frac{3}{4}} + 12x^{-5}$

3. (9 pts) a. Find the slope of the secant line through the points $(-1, f(-1))$ and $(-1+h, f(-1+h))$ for the function

$$f(x) = \frac{8-2x}{x+3}.$$

$f(-1) = 5$

$$\frac{f(-1+h) - f(-1)}{h} = \frac{1}{h}\left[\frac{10-2h}{2+h} - \frac{10}{2}\right] = \frac{1}{h}\left[\frac{20-4h-(20+10h)}{2(2+h)}\right] = \frac{-14}{2(2+h)}$$

Slope of secant line = $-\frac{7}{2+h}$

$(-1, 5)$

b. Let h get small and determine the slope of the tangent line through $(-1, f(-1))$, which gives the value of the derivative of $f(x)$ at $x = -1$. Give the equation of this tangent line.

$$y - 5 = -\frac{7}{2}(x+1)$$

Slope of tangent line = $-7/2$ Tangent line: $y = -7/2\,x + 3/2$

c. Sketch a graph of $f(x)$ giving x and y-intercepts, vertical and horizontal asymptotes, and including the tangent line at $(-1, f(-1))$.

x-intercept 4 and y-intercept $8/3$

Vertical Asymptote: $x = -3$

Horizontal Asymptote: $y = -2$

GRAPH: