Solve the following initial value problems. (Note some problems may use techniques from previous sections.)

1. $\frac{dy}{dt} = 3t^2y, \qquad y(0) = 44.$

2. $\frac{dy}{dt} = 3 - 2y, \qquad y(0) = 5.$

3. $\frac{dy}{dt} = (t+2)e^{-y}, \qquad y(0) = 19.$

4. $\frac{dy}{dt} = 3t^2y^2, \qquad y(0) = 5.$

5. $\frac{dy}{dt} = \frac{(1-2t)}{2y}, \qquad y(0) = 4.$

6. $\frac{dy}{dt} = 3t^2 + 12, \qquad y(0) = 8.$

7. $\frac{dy}{dt} = 2ty, \qquad y(0) = 5.$

8. $(1+2y)\frac{dy}{dt} = 2t, \qquad y(2) = 0.$

9. $t\frac{dy}{dt} = 2y, \qquad y(1) = 4.$

10. $\frac{dy}{dt} = 2\cos(2t)y^2, \qquad y(0) = 1.$

11. A mothball with volume $V(t)$ has an initial volume$V(0) = 8$ cm^3. The mothball slowly evaporates according to the differential equation

$$\frac{dV}{dt} = -kV^{2/3}.$$

After 3 months, the volume of the mothball has decreased to 1 cm^3, *i.e.*, $V(3) = 1$.

a. Solve this differential equation and find k.

b. Find how long it takes for the mothball to disappear. You should sketch a graph of $V(t)$.

12. Suppose that an empirical study of the growth of a yeast cell indicates that the volume of the cell satisfies the differential equation

$$\frac{dV}{dt} = 0.04V^{3/4}, \qquad V(0) = 1,$$

with V in μm^3 and t in min.

a. Solve this initial value problem.

b. Find how long it takes for the cell to double its volume.

13. a. A population of yeast is growing according to a Malthusian growth model. Suppose that it satisfies the initial value problem

$$\frac{dY}{dt} = 0.08\,Y, \qquad Y(0) = 2000,$$

where t is in hours. Solve this differential equation and determine how long it takes for this population to double.

b. Because of competition from another organism in the broth, the yeast has dwindling supplies of food for growth. An approximate model with a time varying growth rate from this competition is given by the following:

$$\frac{dY}{dt} = (0.08 - 0.002t)Y, \qquad Y(0) = 2000.$$

Solve this differential equation.

c. Find the maximum of this population and when this occurs. Also, determine when the population returns to 2000. You should sketch a graph for this population.

14. Most of the Western European countries are having a dramatic decline in their growth rate to the point where their populations will actually begin to decline early in this century. Consider the case of Austria. Its population was 6.94 million in 1950, 7.47 million in 1970, and 7.72 million in 1990.

a. Use the nonautonomous Malthusian growth model given by

$$\frac{dP}{dt} = (b - at)P, \qquad P(0) = 6.94.$$

Let t be the number of years after 1950, then solve this differential equation, where this solution includes the parameters a and b along with the independent variable t. Use the population data for Austria to find the constants a and b.

b. The population for Austria was 8.13 million in 2000. Use the model above to estimate the population of Austria, then compute the percent error from the actual census data.

c. When does the model predict that Austria will have its largest population (value of t) and what is that population?

15. For many years, the population of India has accelerated in its growth to where soon India will be the world's most populous country. The population of India in 1941 was 319 million, in 1951 it was 361 million, and in 1961 it was 438 million.

a. Consider a Malthusian growth model of the form:

$$\frac{dP}{dt} = rP, P(0) = 319,$$

where t is the number of years after 1941. Use the data in 1941 and 1961 to find the growth constant r for India's population and find the solution for the Malthusian growth model. Find the percent error between this model and the actual data in 1951.

b. Consider the nonautonomous Malthusian growth model given by the differential equation

$$\frac{dP}{dt} = (at + b)P, \qquad P(0) = 319.$$

where the constants a and b are to be determined by the data from the years 1941, 1951, and 1961. Solve this differential equation with the data above, where this solution includes the parameters a and b along with the independent variable t. Use the population data for India to find the constants a and b.

c. If the population of India was 846 million in 1991, then use each of these models to estimate the population in 1991 (in millions) and determine the error between the models and the actual census values. Which model provides the better estimate?