Find the derivative for each of the following functions.

1. $f(x) = x^4 + 7x^3 - 2x^2 - 4x + 3$, 2. $h(t) = t^3 - 5t + \frac{1}{2} - \frac{1}{t^2}$,

3. $p(z) = z^{\frac{1}{3}} + 4.7z^2 - 7\sqrt{z^5}$, 4. $q(w) = 3w^{-0.4} + 2.1w^5 - \frac{2}{\sqrt{w}}$.

5. $g(x) = A - \frac{B}{x^3} + \frac{C}{\sqrt{x}} - Dx^4$.

6. In the linear section, we found that the growth of a child satisfies the equation

$$h(a) = 6.46a + 72.3,$$

where the age, a, is in years and the height, h, is in cm.

a. Find $\frac{dh}{da}$. What is the growth rate at age 2? At age 6?

b. If a child is 135 cm at age 10, what is the predicted height at age 11?

7. The number of species of herpatofauna, N, on Caribbean Islands as a function of the area in square miles, A, is approximated by the formula

$$N = 3A^{\frac{1}{3}}$$

Find the rate of change in number of species as a function of area, $\frac{dN}{dA}$, when the area of the island is 64, 125, and 1000 square miles. You should sketch a graph of the derivative, $\frac{dN}{dA}$, for $0 \leq A \leq 1000$.

8. A ball that is thrown vertically falling under the influence of gravity without air resistance from a 128 ft platform with an upward velocity of 32 ft/sec satisfies the equation

$$h(t) = 128 + 32t - 16t^2,$$

where h is in feet and t is in seconds.

a. Find an expression for the velocity, $v(t) = h'(t)$. Determine when the velocity is zero, then determine the maximum height of the ball. What is the velocity at $t = 2$ and $t = 4$?

b. You should sketch a graph of $h(t)$, showing crucial points, including the h-intercept, the maximum height, and determine when the ball hits the ground.

9. A cat is crouching on a ledge that is 12 feet above the ground, trying to ambush pigeons that fly by.

a. Suppose that a pigeon flies by 4 feet above the cat, and that the cat jumps off the ledge with just enough vertical velocity, V to catch the pigeon. If the height of the cat is given by

$$h(t) = -16t^2 + Vt + 12,$$

then find the velocity $v(t) = h'(t)$ of the cat at any time, $t \geq 0$.

b. Find when the velocity is equal to zero in terms of V. This is the time at the maximum height.

c. Since the cat is 16 ft in the air at this time, use the equation for the height of the cat, $h(t)$, to compute the initial velocity of the cat, V. Substitute this into the velocity equation, $v(t)$, to give the velocity of the cat after 1 second.

d. Find when the cat hits the ground with the pigeon and find the velocity of the cat the instant before it hits the ground.

10. Suppose that a population of insects, P (individuals), in a controlled experiment with constant food supply is shown to have a growth rate that fits the logistic growth function

$$g(P) = 0.04P\left(1 - \frac{P}{800}\right),$$

where the time is measured in days.

a. Find the population when the growth rate $g(P)$ is zero (the P-intercepts). This gives the equilibria for this experiment. Think about a biological interpretation for each of the equilibria.

b. Compute the derivative of $g(P)$, then determine the population where $g(P)$ has a maximum growth rate, and what the maximum growth rate is. You should think about the biological interpretation of this maximum growth rate, and sketch the graph of $g(P)$.

11. a. Lizards are cold-blooded animals whose temperatures roughly match the surrounding environment. Suppose the body temperature, $T(t)$, of a lizard is measured for a period of 18 hours from midnight until 6 PM. The body temperature (in $^\circ C$) of the lizard over this period of time (in hours) is found to be well approximated by the polynomial

$$T(t) = -0.01t^3 + 0.285t^2 - 1.80t + 15.$$

Find the general expression for the rate of change of body temperature per hour $\left(\frac{dT}{dt}\right)$.

b. Use this information to find what the rate of change of body temperature is at midnight, 4 AM, 8 AM, noon, and 4 PM. Which of these times gives the fastest increase in the body temperature and which shows the most rapid cooling of the lizard?