

Find the derivatives of the following functions:

$$1. f(x) = \frac{x^3 - \ln(x)}{1 - x^2} + \frac{2}{x^2},$$

$$2. f(x) = \frac{x^2 - e^{-x}}{3x + 1} + xe^{-x},$$

$$3. f(x) = \frac{\sqrt{x}}{2 + x} - \frac{1}{e^{3x}},$$

$$4. f(x) = \frac{8e^{-2x}}{12 + \cos(2x)}.$$

Find the derivative of the following functions. Find the  $x$  and  $y$ -intercepts and any asymptotes if they exist. Find the critical points and determine if they are relative maxima or minima for each graph. You should sketch the curves of the functions.

$$5. y = \frac{x^2}{x + 1},$$

$$6. y = \frac{e^x}{x + 1},$$

$$7. y = \frac{x^2 - 2x + 2}{x - 1}.$$

8. Consider the function

$$y = \frac{1}{\sin(3x)}.$$

Find the derivative and the period  $T$  of the function. Find the critical points for  $x \in [0, T]$  and determine if they are relative maxima or minima. Find the three vertical asymptotes for  $x \in [0, T]$ . You should sketch the curve of the function.

9. Carbon monoxide (CO) binds about 200 times more effectively than oxygen to hemoglobin, forming the complex called carboxyhemoglobin. This tight binding affinity makes death by carbon monoxide poisoning a problem in industrial settings as well as from running cars in a garage. Let  $p$  be the partial pressure of CO measured in torrs, then a dissociation curve for CO and hemoglobin is given by

$$y(p) = \frac{p^4}{0.0625 + p^4},$$

where  $y$  is the fraction of hemoglobin bound by CO.

a. Differentiate  $y(p)$  and also find the second derivative,  $y''(p)$ . Find the value of  $p$  that satisfies  $y''(p) = 0$ . Give the  $p$  and  $y$  values for this point of inflection ( $p > 0$ ). Find the value of  $y'(p)$  at this point of inflection.

b. Find any intercepts and asymptotes for  $y(p)$ . You should sketch a graph of  $y(p)$ .

c. Find the partial pressure of CO that results in the hemoglobin being 90% saturated. Compare this dissociation curve to the one in the lecture notes.

10. Jacob and Monod developed the theory of genetic control by induction. This is a very important control made most famous by the *lac* operon. The enzyme  $\beta$ -galactosidase is induced to catalyze the break down of lactose into simple sugars (glucose and galactose) for energy in the cell. The rate of induction for  $\beta$ -galactosidase is given by the formula

$$R(L) = \frac{V_{max}L^2}{K + L^2},$$

where  $L$  is the concentration of lactose and  $V_{max}$  and  $K$  are kinetic constants.

a. Suppose that  $V_{max} = 10$  and  $K = 1$ . Differentiate this rate function and also find its second derivative. Give both the  $L$  and  $R$  values for any points of inflection, ( $L \geq 0$ ), and the value of  $R'(L)$  at the point of inflection.

b. Find any intercepts and asymptotes for  $R(L)$ . You should then sketch a graph of  $R(L)$ .

c. For  $R'(L)$ , find any intercepts, asymptotes, and extrema (where  $R''(L) = 0$ ). You should then sketch the graph of  $R'(L)$ . Relate the maximum here to the point of inflection in Part a.

11. Some entomologists use Hassell's model for studying the population of insects. An updating function that gives the population of the insects in the next generation (or time period) is given by

$$H(P) = \frac{5P}{1 + 0.004P}.$$

where  $P$  is the current population of insects.

a. Differentiate this function, then find the second derivative. Is  $H''(P)$  positive or negative for  $P \geq 0$ ?

b. Find any intercepts and asymptotes for  $H(P)$  ( $P \geq 0$ ). You should sketch a graph of  $H(P)$  using this information.

c. Equilibria are found by solving

$$P_e = H(P_e).$$

Find all equilibria of the model, that is, what populations remain constant for any time period.

12. The logistic growth model is one of the most common models used in ecological research. Consider a yeast population that satisfies the logistic growth model

$$Y(t) = \frac{1000}{1 + 19e^{-0.1t}},$$

where  $t$  is in hours and  $Y$  is in yeast/cc.

a. Differentiate this function, then find the second derivative. Use the second derivative to find any points of inflection ( $t \geq 0$ ), giving both the  $t$  and  $Y$  values.

b. Find any intercepts and asymptotes for  $Y(t)$  with  $t \geq 0$ . You should sketch a graph of  $Y(t)$ .

c. Find how long it takes for the initial population to double.

d. A Malthusian growth model that approximates this yeast population during the early stages of growth is given by

$$M(t) = 50e^{0.1t}.$$

Find a doubling time from this model, then compare your result to that of the logistic growth model above.