1. This equation can be factored $(x+7)(x-3) = 0$ so the roots are $x_1 = -7$, $x_2 = 3$.

2. This equation can be factored $(x-8)(x-1) = 0$ so the roots are $x_1 = 1$, $x_2 = 8$.

3. This equation can be factored $(2x-9)(x+2) = 0$ so the roots are $x_1 = -2$, $x_2 = \frac{9}{2}$.

4. This equation cannot be factored, so we apply the quadratic formula and obtain complex roots:

$$x = \frac{-1 \pm \sqrt{1-20}}{2} = -\frac{1}{2}\left(1 \pm i\sqrt{19}\right).$$

5. This equation cannot be factored. so we apply the quadratic formula and obtain:

$$x = \frac{2 \pm \sqrt{4+28}}{2} = 1 \pm \sqrt{8}.$$

6. For the line, the y-intercept is $(0, 2)$, and the slope is $m = 2$. The x-intercept solves $0 = 2x+2$, so $x = -1$ and the x-intercept is $(-1, 0)$.

For the parabola, the y-intercept is $(0, -5)$. The x-intercepts solve

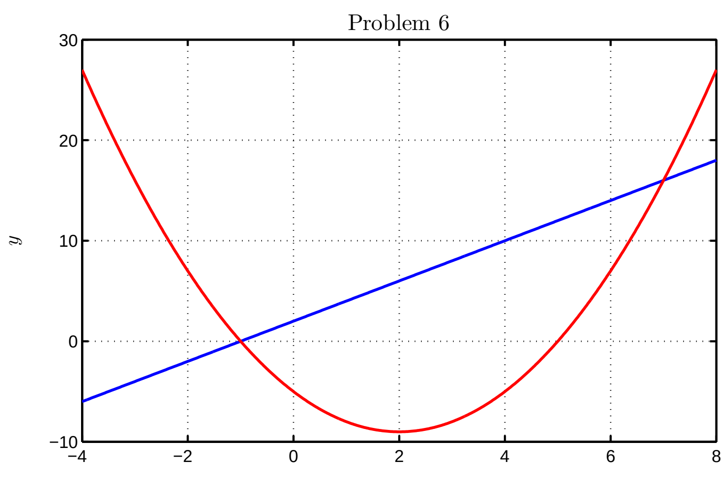
$$x^2 - 4x - 5 = (x-5)(x+1) = 0 \quad \text{or} \quad x = -1, 5.$$

Thus, the x-intercepts are $(-1, 0)$ and $(5, 0)$. The x value of the vertex is the midpoint between the intercepts, so $x_v = \frac{-1+5}{2} = 2$. Since $g(2) = -9$, the vertex is $(2, -9)$.

The points of intersection satisfy $f(x) = g(x)$ or $2x + 2 = x^2 - 4x - 5$, so

$$x^2 - 6x - 7 = (x+1)(x-7) = 0.$$

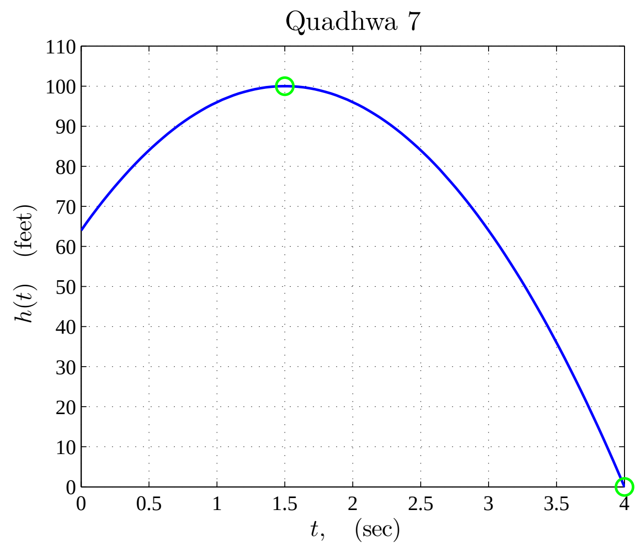
When $x = -1$, then $f(-1) = 0$, so this point of intersection is $(-1, 0)$. When $x = 7$, then $f(7) = 16$, so this point of intersection is $(7, 16)$. The graph is below



7. The height of the ball can be written in factored form

$$h(t) = -16t^2 + 48t + 64 = -16(t^2 - 3t - 4) = -16(t+1)(t-4)$$

From this factored form, it is easy to see that the ball hits the ground at $t = 4$ sec. The vertex is the midpoint of the t-intercepts, so $t = \frac{4-1}{2} = 1.5$ sec with $h(1.5) = 100$ ft. Below is the graph of the height of the ball.



8. From the lecture notes, we have

$$[\mathrm{H}^+] = \frac{1}{2}\left(-K_a + \sqrt{K_a^2 + 4K_a x}\right),$$

where x is the normality of the solution. For a 0.1N solution, $x = 0.1$, so

$$[\mathrm{H}^+] = \frac{1}{2}\left(-1.75 \times 10^{-5} + \sqrt{(1.75 \times 10^{-5})^2 + 0.4(1.75 \times 10^{-5})}\right) = 0.001314.$$

It follows that the pH $= -\log_{10} 0.001314 = 2.881$.

Similarly, for a 1N solution, $x = 1$, so

$$[\mathrm{H}^+] = \frac{1}{2}\left(-1.75 \times 10^{-5} + \sqrt{(1.75 \times 10^{-5})^2 + 4(1.75 \times 10^{-5})}\right) = 0.004175.$$

It follows that the pH $= -\log_{10} 0.004175 = 2.379$.

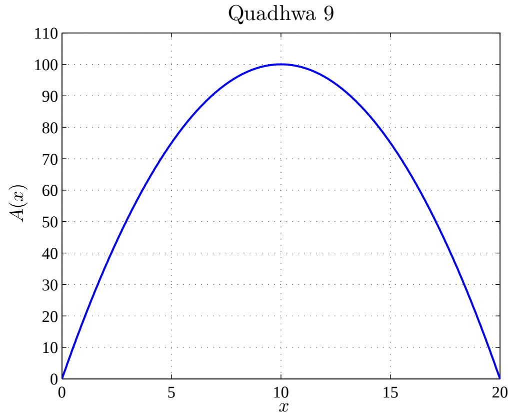
9. a. The perimeter of the rectangle is 40 cm, so $2y + 2x = 40$. It follows that $y = 20 - x$.

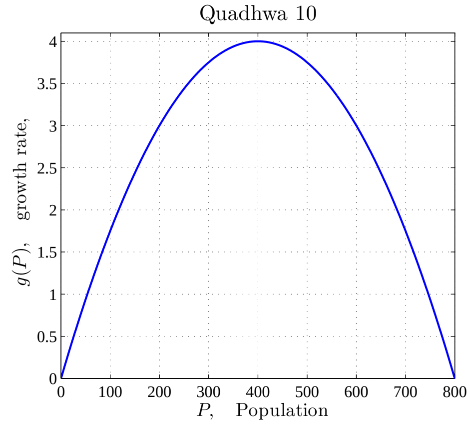
b. The area of a rectangle is $A = xy$, so

$$A(x) = 20x - x^2.$$

c. The domain of $A(x)$ is $0 < x < 20$. The maximum area occurs at the vertex of the parabola shown below on the left, so $x = 10$ and $A_{max} = 100$ cm^2. Thus, the rectangle with the maximum area is a square.

d. $A(x)$ is a parabola. The graph is shown below on the left.





10. a. The equilibrium population satisfies

$$g(P_e) = 0.02P_e - 0.000025P_e^2 = 0.02P_e(1 - 0.00125P_e) = 0,$$

so $P_e = 0$ or $P_e = \frac{1}{0.00125} = 800$ individuals.

b. The maximum growth rate occurs at the vertex of parabola, which satisfies $P = 400$, the midpoint between the P-intercepts (or equilibria) of the growth function. Thus, the maximum growth rate is $g(400) = 0.02(400) - 0.000025(400)^2 = 4$ individuals per generation. A sketch of the graph $g(P) = 0.02P - 0.000025P^2$ is shown above on the right.