1. Consider the function $f(x) = (x^3 - 3x^2 + 7)(x^4 - 2x^2 + 6x - 1)$. Applying the product and power rules, we find the derivative:

$$f'(x) = (x^3 - 3x^2 + 7)(4x^3 - 4x + 6) + (3x^2 - 6x)(x^4 - 2x^2 + 6x - 1).$$

2. Consider the function $f(x) = e^{2x}\cos(3x)$. Applying the product rule, we find the derivative:

$$f'(x) = -3e^{2x}\sin(3x) + 2e^{2x}\cos(3x) = e^{2x}(2\cos(3x) - 3\sin(3x)).$$

3. Consider the function $f(x) = x^2e^{-x} + 21\sqrt{x} = x^2e^{-x} + 21x^{\frac{1}{2}}$. Applying the product rule, we find the derivative:

$$f'(x) = -x^2e^{-x} + 2xe^{-x} + 21\left(\frac{1}{2}\right)x^{-\frac{1}{2}} = (2x - x^2)e^{-x} + \frac{21}{2\sqrt{x}}.$$

4. Consider the function $f(x) = \frac{1}{x^2}\ln(x) - e^{2x}(x^2 - 1) = x^{-2}\ln(x) - e^{2x}(x^2 - 1)$. Applying the product rule, we find the derivative:

$$f'(x) = x^{-2}\frac{1}{x} - 2x^{-3}\ln(x) - 2xe^{2x} - 2e^{2x}(x^2 - 1) = \frac{1}{x^3}(1 - 2\ln(x)) - 2e^{2x}(x^2 + x - 1)$$

5. Consider the function

$$f(x) = \frac{20\cos(7x)}{x^5} + (5x^5 + 11)\sin(2(x - 18)) = 20x^{-5}\cos(7x) + (5x^5 + 11)\sin(2(x - 18)).$$

Applying the product rule, we find the derivative:

$$f'(x) = -140x^{-5}\sin(7x) - 100x^{-6}\cos(7x) + 2(5x^5 + 11)\cos(2(x - 18)) + 25x^4\sin(2(x - 18)).$$

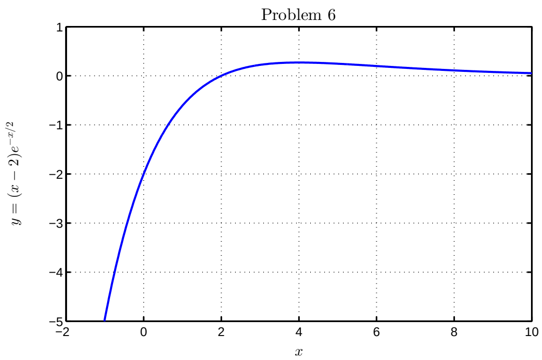
6. Consider the function $y = (x - 2)e^{-x/2}$. We find the derivative

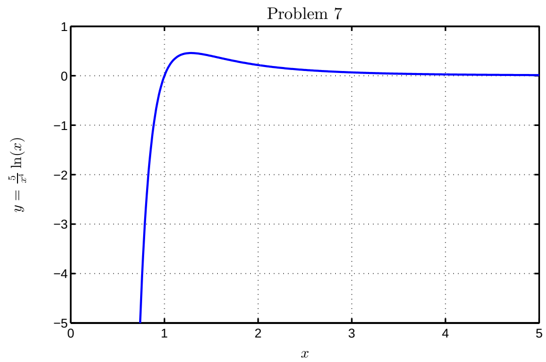
$$y'(x) = -\frac{1}{2}(x - 2)e^{-x/2} + 1\cdot e^{-x/2} = \left(\frac{4 - x}{2}\right)e^{-x/2}.$$

The x-intercept is where $y = 0$, so $x = 2$. The y-intercept occurs where $x = 0$, so $y = -2$. Since the exponential function goes to zero faster than the term $(x - 2)$,

$$\lim_{x\to+\infty}(x - 2)e^{-x/2} = 0,$$

so there is a horizontal asymptote at $y = 0$ to the right. The critical point satisfies $y'(x) = 0$, so $x_c = 4$, which gives $y(x_c) = (4 - 2)e^{-2} \approx 0.2707$. This is a relative maximum at $(4, 2e^{-2})$. The graph is shown below to the left.





7. Consider the function $y = \frac{5}{x^4}\ln(x) = 5x^{-4}\ln(x)$. We find the derivative

$$y'(x) = 5x^{-4}\left(\frac{1}{x}\right) - 20x^{-5}\ln(x) = 5x^{-5}(1 - 4\ln(x)).$$

The domain is $0 < x < +\infty$, and there is a vertical asymptote where $x = 0$. Thus, there is no y-intercept. The x-intercept occurs where $y = 0$, so $\ln(x) = 0$ or $x = 1$. Since x^{-4} goes to zero faster than the term $\ln(x)$,

$$\lim_{x\to+\infty} 5x^{-4}\ln(x) = 0,$$

so there is a horizontal asymptote at $y = 0$ to the right. There is a critical point where $y'(x) = 0$, or $1 - 4\ln(x) = 0$. Thus, $x_c = e^{\frac{1}{4}} \approx 1.28403$ and $y(x_c) \approx 5(1.28403)^{-4}\ln(1.28403) \approx 0.45985$. This is a relative maximum at $(e^{1/4}, \frac{5}{4}e^{-1})$. The graph is shown above to the right.

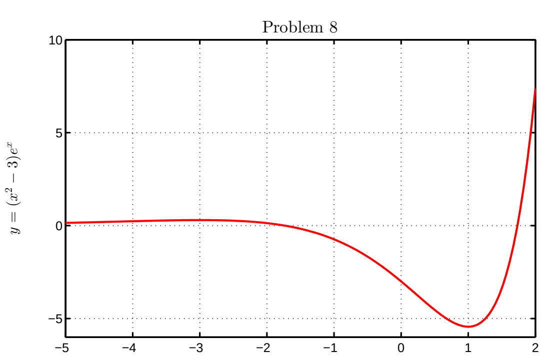
8. Consider the function $y = (x^2 - 3)e^x$. We find the derivative

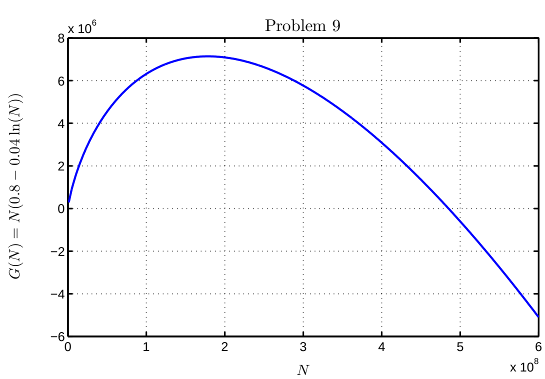
$$y'(x) = (x^2 - 3)e^x + (2x)e^x = (x^2 + 2x - 3)e^x = (x + 3)(x - 1)e^x.$$

The x-intercepts occur where $y = 0$, so $x^2 - 3 = 0$ or $x = \pm\sqrt{3}$. The y-intercept occurs where $x = 0$, so $y = -3$. Since the exponential function goes to zero faster than the term $(x^2 - 3)$, as $x \to -\infty$,

$$\lim_{x\to-\infty} (x^2 - 3)e^x = 0,$$

so there is a horizontal asymptote at $y = 0$ to the left. There are critical points where $y'(x) = 0$ or $(x + 3)(x - 1) = 0$. Thus, $x_{1c} = -3$ and $x_{2c} = 1$. Then $y(x_{1c}) = (9 - 3)e^{-3} \approx 0.298722$ is a relative maximum at $(-3, 6e^{-3})$ and $y(x_{2c}) = (1 - 3)e^1 \approx -5.4366$ is a relative minimum at $(1, -2e^1)$. The graph is shown below to the left.





9. a. The growth function is $G(N) = N(0.8 - 0.04\ln(N))$. The equilibrium satisfies $N_e(0.8 - 0.04\ln(N_e)) = 0$. Since $N = 0$ is not in the domain, the equilibrium satisfies $0.04\ln(N_e) = 0.8$ or $\ln(N_e) = 20$. It follows that the equilibrium is $N_e = e^{20} \approx 4.8517 \times 10^8$.

b. By the product rule, the derivative is

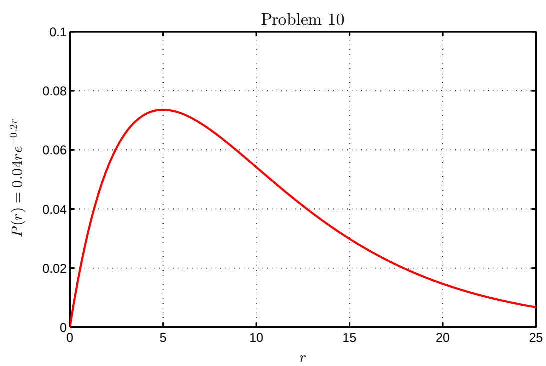
$$G'(N) = N\left(\frac{-0.04}{N}\right) + (0.8 - 0.04\ln(N)) = 0.76 - 0.04\ln(N).$$

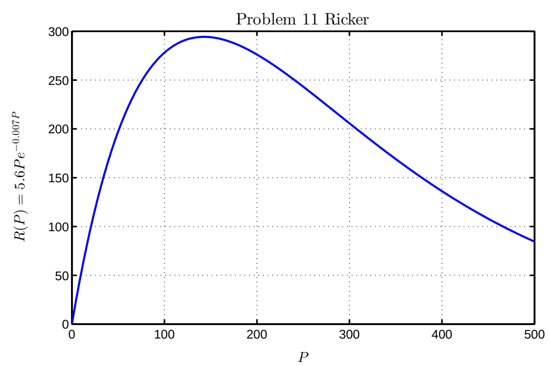
The maximum growth rate satisfies $0.76 - 0.04\ln(N) = 0$ or $\ln(N) = 19$. Thus, the maximum rate of growth occurs at $N_{max} = e^{19} = 1.785 \times 10^8$ with a maximum growth rate of $G(N_{max}) = 7.139 \times 10^6$. The graph is shown above to the right.

10. Consider the function $P(r) = 0.04re^{-0.2r}$. By the product rule, the derivative is $P'(r) = 0.4(-0.2re^{-0.2r} + e^{-0.2r}) = 0.04(1 - 0.2r)e^{-0.2r}$. The maximum probability occurs when the derivative is zero, thus

$$0.04(1 - 0.2r)e^{-0.2r} = 0 \quad \text{or} \quad 0.2r = 1.$$

Thus, the maximum probability of a seed landing occurs at $r_{max} = 5$ m with a probability of $P(5) = 0.073576$. The graph of the probability density function has an intercept at $(0, 0)$, $(P(0) = 0)$. Since the exponential $(e^{-0.2r})$ dominates the linear part (r), we find $\lim_{r\to+\infty} P(r) = 0$. Thus, there is a horizontal asymptote of $P = 0$ (as $r \to +\infty$) and a local maximum of $(5, 0.073576)$. The graph is shown below to the left.





11. a. The model becomes $P_{n+1} = R(P_n) = 5.6P_ne^{-0.007P_n}$, so the equilibria satisfy $P_e = 5.6P_ee^{-0.007P_e}$. One solution is the extinction equilibrium, $P_e = 0$. The other equilibrium satisfies $1 = 5.6e^{-0.007P_e}$ or $e^{0.007P_e} = 5.6$. It follows that the carrying capacity equilibrium is $P_e = \frac{\ln(5.6)}{0.007} \approx 246.1095$.

b. The derivative of $R(P)$ is $R'(P) = 5.6(-0.007Pe^{-0.007P} + 1 \cdot e^{-0.007P}) = 5.6(1 - 0.007P)e^{-0.007P}$.

c. Evaluating at $P = 110$, we see $R'(110) = 5.6(1 - 0.007(110))e^{-0.007(110)} \approx 0.59636$. At $P = 770$, $R'(770) = 5.6(1 - 0.007(770))e^{-0.007(770)} \approx -0.11215$.

d. For the updating function, $R(P) = 5.6P_ne^{-0.007P_n}$, the P intercept occurs where $R(P) = 0$, so $0 = 5.6Pe^{-0.007P}$, or $P = 0$. The $R(P)$ intercept is therefore also at 0. It is typical that a closed discrete population model passed through the origin (the extinction equilibrium). The maximum value of $R(P)$ occurs where $R'(P) = 0$ so $P_{max} = \frac{1}{0.007} \approx 142.857$. Then

$R(P_{max}) = \left(\frac{5.6}{0.007}\right) e^{-\frac{0.007}{0.007}} \approx 294.303$. Since the exponential function dominates, there is a horizontal asymptote at $R = 0$. The graph is shown above to the right.

12. a. Consider the fishery management model with the growth function $R(P) = 5Pe^{-0.002P} - 1.5P$, where P is in fish/100 m of river and $R(P)$ has units of fish/100 m/day. The equilibrium population of the fish in when the growth is zero or when $R(P) = 0$. Thus, $5Pe^{-0.002P} - 1.5P = P(5e^{-0.002P} - 1.5) = 0$, so clearly one equilibrium is the extinction equilibrium, $P_e = 0$. The carrying capacity satisfies $5e^{-0.002P_e} = 1.5$ or $e^{0.002P_e} = \frac{5}{1.5}$ or $P_e = 500\ln\left(\frac{5}{1.5}\right) \approx$ 601.99 fish/100 m of river.

b. The derivative of the growth function is

$$R'(P) = 5(-0.002Pe^{-0.002P} + e^{-0.002P}) - 1.5 = 5(1 - 0.002P)e^{-0.002P} - 1.5.$$

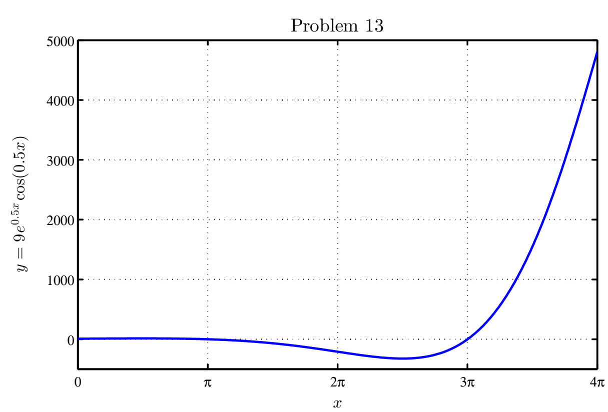
c. We evaluate this derivative at $P = 200$, 250, and 300, giving:
$R'(200) = 5(1 - 0.4)e^{-0.4} - 1.5 \approx 0.51096$
$R'(250) = 5(1 - 0.5)e^{-0.5} - 1.5 \approx 0.016327$
$R'(300) = 5(1 - 0.6)e^{-0.6} - 1.5 \approx -0.40238$

d. With the fishing model $R(P) = 5Pe^{-0.002P} - (h+1)P$, we want to find the minimum level of fishing h that guarantees extinction of the fish. This is found by having the carrying capacity fall to zero or $5e^{-0.002P_e} = 1 + h$ with $P_e = 0$. Thus, $5 = 1 + h$ or $h = 4$.

13. Consider the function $y = 9e^{0.5x}\cos(0.5x)$. The derivative satisfies
$y'(x) = 9(-0.5e^{0.5x}\sin(0.5x) + 0.5e^{0.5x}\cos(0.5x)) = 4.5e^{0.5x}(\cos(0.5x) - \sin(0.5x))$. The critical points occur where $y'(x) = 0$, so $\cos(0.5x) = \sin(0.5x)$. Sine and cosine are equal when

$$0.5x = \frac{\pi}{4}, \ \frac{5\pi}{4},$$

so $x_{1c} = \frac{\pi}{2}$ with $y(x_{1c}) = 9e^{0.25\pi}\cos(0.25\pi) \approx 13.9579$ which is a relative maximum, and $x_{2c} = \frac{5\pi}{2}$ with $y(x_{2c}) = 9e^{1.25\pi}\cos(1.25\pi) \approx -322.997$, a relative minimum. For $x \in [0, 4\pi]$, the endpoints are $y(0) = 9$ and $y(4\pi) = 9e^{2\pi}\cos(2\pi) \approx 4819.4$. It follows that this is the absolute maximum in this interval, while the relative minimum above is the absolute minimum in this interval. The graph is below.



14. The velocity of air passing through the windpipe satisfies:

$$v(r) = Ar^2(R - r) = ARr^2 - Ar^3,$$

where A and R are constants. Differentiating $v(r)$ gives $v'(r) = 2ARr - 3Ar^2 = Ar(2R - 3r)$. Thus, critical points occur at $r_c = 0$ and $2R - 3r_c = 0$ or $r_c = \frac{2R}{3}$. The former is clearly a minimum, so the maximum air velocity occurs at $r_{max} = \frac{2R}{3}$ with a maximum air velocity of

$$v(r_{max}) = A\left(\frac{2R}{3}\right)^2\left(R - \frac{2R}{3}\right) = \frac{4AR^3}{27}.$$

15. a. Consider the damped oscillator given by $h(t) = 46e^{-0.07t}\sin(4t)$. We want to satisfy $h(t_0) = 0$ $(t_0 > 0)$. Thus, we need $\sin(4t) = \sin(\pi)$ or $t_0 = \frac{\pi}{4}$.

b. The derivative of $h(t)$ satisfies

$$h'(t) = 46(4e^{-0.07t}\cos(4t) - 0.07e^{-0.07t}\sin(4t)) = 46e^{-0.07t}(4\cos(4t) - 0.07\sin(4t)).$$

c. For the domain $t \geq 0$ with $h(0) = 0$, the exponential decay in $h(t)$ means that the absolute maximum is the first maximum. The first maximum solves $h'(t) = 0$ or

$$4\cos(4t) = 0.07\sin(4t) \qquad \text{or} \qquad \tan(4t) = \frac{4}{0.07} \approx 57.1429,$$

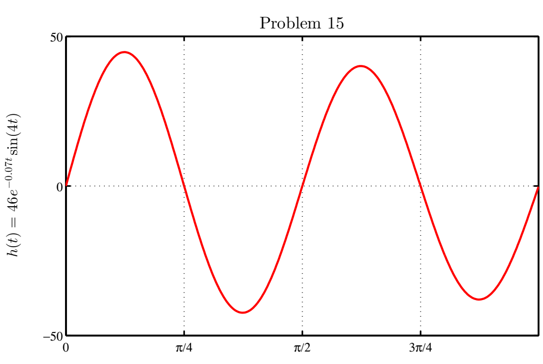
$$t_{max} = \frac{1}{4}\arctan\left(\frac{4}{0.07}\right) \approx 0.38832.$$

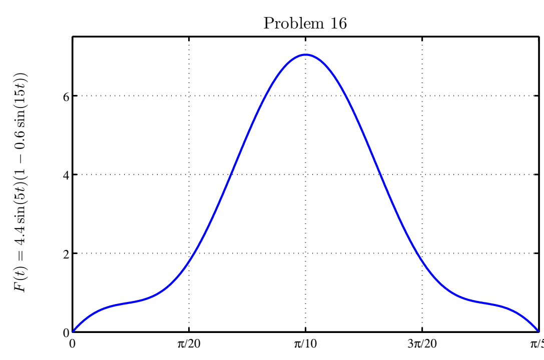
Substituting this into h gives $h(t_{max}) = 46e^{-0.07t_{max}}\sin(4t_{max}) \approx 44.7596$.

d. Similarly, the absolute minimum is the first minimum. This minimum occurs half a period after the maximum. The period of $\sin(4t)$ is $\pi/2$. Thus,

$$t_{min} = \frac{1}{4}\arctan\left(\frac{4}{0.07}\right) + \frac{\pi}{4} \approx 1.17372.$$

Substituting this into h gives $h(t_{min}) = 46e^{-0.07t_{min}}\sin(4t_{min}) \approx -42.3652$. The graph is below to the left.





16. a. Consider the approximating force function $F(t) = 4.4\sin(5t)(1 - 0.6\sin(15t))$. The force is zero at t = 0. We note that $1 - 0.6\sin(15t)$ is never zero, so the next time when the force is zero is when $\sin(5t) = 0$ or $t = \frac{\pi}{5}$. This the length of time that the foot is on the ground.

b. The derivative of $F(t)$ is

$$F'(t) = 4.4(\sin(5t)(-9\cos(15t)) + 5\cos(5t)(1 - 0.6\sin(15t))).$$

At $t_m = \frac{\pi}{10}$, we see $\cos(15t_m) = \cos(5t_m) = 0$. It follows that $F'(t_m) = 0$, and the functional value gives this to be a relative maximum. Substituting, we find $F(t_m) = 4.4(1)(1+0.6) = 7.04$. The graph is above to the right.