Find the derivatives of the following functions:

1. $f(x) = (x^3 - 3x^2 + 7)(x^4 - 2x^2 + 6x - 1)$,
2. $f(x) = e^{2x}\cos(3x)$,
3. $f(x) = x^2 e^{-x} + 21\sqrt{x}$,
4. $f(x) = \frac{1}{x^2}\ln(x) - e^{2x}(x^2 - 1)$,
5. $f(x) = \dfrac{20\cos(7x)}{x^5} + (5x^5 + 11)\sin(2(x - 18))$.

Find the derivative and sketch the curves of the functions below. Give the domain of each of the functions. List all maxima and minima for each graph. Also, give the x and y-intercepts and any asymptotes if they exist. You should sketch the graphs on separate paper.

6. $y = (x - 2)e^{-x/2}$,
7. $y = \frac{5}{x^4}\ln(x)$,
8. $y = (x^2 - 3)e^x$.

9. A tumor growing according to Gompertz's model satisfies the growth law

$$G(N) = N(0.8 - 0.04\ln(N)) \text{ (cells/day)},$$

where N is the number of tumor cells and the time units are days.

a. Find the equilibrium number of tumor cells by solving when $G(N) = 0$. Note that unlike most population models, $N = 0$ is not an equilibrium because it is outside the domain of the function.

b. Compute $G'(N)$ and determine the population at which the maximum rate of growth of the tumor is occurring. What is the maximum growth rate (in cells/day)? You should make a sketch of this graph using the information you found above.

10. The distribution of seeds from around a plant satisfies an exponential distribution. It is more likely to find seeds close to a plant than it is far away. Suppose that experimental measurements fitting the seed distribution radially from the plant satisfy

$$P(r) = 0.04re^{-0.2r},$$

where P is the probability of finding a seed r meters from the plant. Find the distance r and the probability $P(r)$ at which a seed is most likely to land. That is, find the maximum probability from the function above. Evaluate $P(0)$. Find the horizontal asymptote by evaluating $\lim_{r\to\infty} P(r)$. Sketch a graph of $P(r)$.

11. Many biologists in fishery management use Ricker's model to study the population of fish. Let P_{n+1} be the population of fish in any year n, then Ricker's model is given by

$$P_{n+1} = R(P_n) = aP_n e^{-bP_n}.$$

Suppose that the best fit to a set of data gives $a = 5.6$ and $b = 0.007$ for the number of fish sampled from a particular river.

a. Find the equilibrium population of the fish by finding when $P_e = R(P_e)$.

b. Find the rate of change in the growth rate by the derivative of $R(P)$.

c. Evaluate $R'(P)$ at $P = 110.$ and at $P = 770.$

d. Consider the updating function $R(P)$. We would like to create a graph of this function. Find the P and $R(P)$ intercepts. Find the maximum value of $R(P)$ and the value of P at which it occurs. Find the horizontal asymptote of R.

12. Many biologists in fishery management use Ricker's model to study the population of fish. A general expression for the growth of the population of fish is given by

$$R(P) = aPe^{-bP} - (h+1)P,$$

where a and b are parameters that fit the population dynamics of the fish and h is the harvesting level of the fisheries. Suppose that the best fit to a set of data for the number of fish sampled from a particular river gives

$$R(P) = 5Pe^{-0.002P} - 1.5P,$$

where P is in fish/100 m of river and $R(P)$ has units of fish/100 m/day.

a. Find the equilibrium population of the fish by finding when $R(P) = 0$.

b. Find the rate of change in the growth rate by the derivative of $R(P)$.

c. Evaluate $R'(P)$ at $P = 200$, 250, and 300. What are the units for these calculations?

d. We noted that the growth of the population was given by $R(P) = 5Pe^{-0.002P} - (h+1)P$, with the fishing level of h. Determine the minimum intensity of fishing, h, that leads to extinction of the fish. (The only equilibrium is zero.)

13. Consider the following function:

$$y = 9e^{0.5x}\cos(0.5x).$$

Find the derivative, $y'(x)$. Find all critical points for $x \in [0, 4\pi]$, and decide if they are relative maxima or minima. Find the absolute maximum for $x \in [0, 4\pi]$. You should make a sketch of this graph.

14. When coughing, the windpipes contract to increase the velocity of air passing through the windpipe to help clear mucus. The velocity, v, at which the air flows through the windpipe depends on the radius, r of the windpipe. If R is the resting radius of the windpipe, then the velocity of air passing through the windpipe satisfies:

$$v(r) = Ar^2(R - r),$$

where A is a constant dependent on the strength of the diaphragm muscles. Find the derivative of the velocity function. Find the value of r that maximizes the velocity of air and determine the maximum velocity of the air flowing through the windpipe.

15. Many applications are modeled with a damped oscillator. Consider the model satisfying:

$$h(t) = 46e^{-0.07t}\sin(4t).$$

a. Find the first time $t_0 > 0$, where $h(t_0) = 0$.

b. Find the derivative of $h(t)$.

c. Find the absolute maximum of $h(t)$ for $t \geq 0$.

d. Find the absolute minimum of $h(t)$ for $t \geq 0$.

16. When a particular person is walking, the magnitude of the vertical force acting on one foot can be approximated by the function

$$F(t) = 4.4 \sin(5t)(1 - 0.6 \sin(15t)),$$

where t is the time in seconds.

a. Clearly, the force is zero at t = 0. Find the next time when the force is zero. This the length of time that the foot is on the ground.

b. Find the derivative of $F(t)$. Evaluate the derivative at $t_m = \frac{\pi}{10}$. With this information, what is this maximum value of $F(t)$?