1. a. We solve $x^3 + 3x^2 - 4x = 0$.

$$\begin{aligned} x\left(x^2 + 3x - 4\right) &= 0 \\ x(x+4)(x-1) &= 0 \\ x &= -4, \quad 0, \quad \text{or} \quad 1. \end{aligned}$$

1. b. We solve $12x^2 - 4x^3 - x^4 = 0$.

$$\begin{aligned} -x^2\left(x^2 + 4x - 12\right) &= 0 \\ x^2(x+6)(x-2) &= 0 \\ x &= -6, \quad 0, \quad \text{or} \quad 2. \end{aligned}$$

2. a. We solve $x - \frac{24}{x+2} = 3$.

$$\begin{aligned} (x+2)\left(x - \frac{24}{x+2}\right) &= 3(x+2), \qquad x \neq -2 \\ x^2 + 2x - 24 &= 3x + 6 \\ x^2 - x - 30 &= 0 \\ (x-6)(x+5) &= 0 \\ x &= -5 \quad \text{or} \quad 6. \end{aligned}$$

2. b. We solve $x - \frac{9}{x} = 0$.

$$\begin{aligned} x\left(x - \frac{9}{x}\right) &= x(0), \qquad x \neq 0 \\ x^2 - 9 &= 0 \\ x &= -3 \quad \text{or} \quad 3 \end{aligned}$$

3. a. We solve $\frac{4}{x^2} - \frac{3}{x} = 1$.

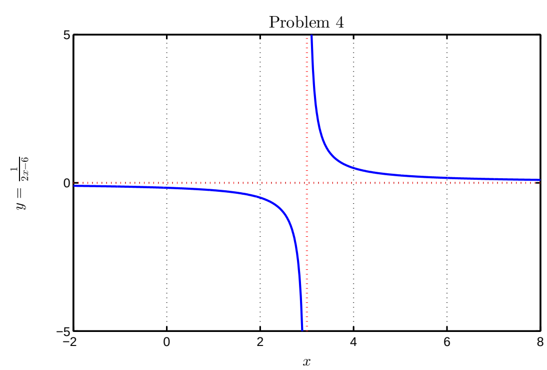
$$\begin{aligned} x^2\left(\frac{4}{x^2} - \frac{3}{x}\right) &= x^2(1), \qquad x \neq 0 \\ 4 - 3x &= x^2 \\ x^2 + 3x - 4 &= 0 \\ (x+4)(x-1) &= 0 \\ x &= -4 \quad \text{or} \quad 1. \end{aligned}$$

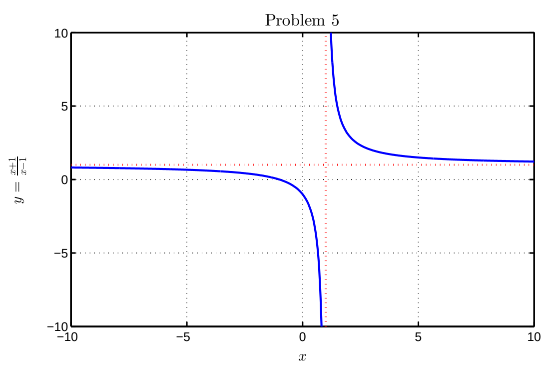
3. b. We solve $x + \frac{6}{x-1} = -5$.

$$
\begin{aligned}
(x-1)\left(x + \frac{6}{x-1}\right) &= (x-1)(-5), \qquad x \neq 1 \\
x^2 - x + 6 &= -5x + 5 \\
x^2 + 4x + 1 &= 0, \qquad \text{Apply Quadratic Formula} \\
x &= \frac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3} \\
x &= -2 - \sqrt{3} \quad \text{or} \quad -2 + \sqrt{3}
\end{aligned}
$$

For each of the following functions, determine the domain. Find any x or y-intercepts and locate any vertical or horizontal asymptotes. Sketch the graphs of the functions.

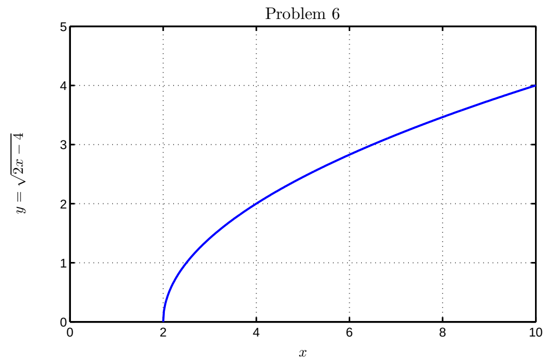
4. For $y = \frac{1}{2x-6}$, the denominator is zero when $2x - 6 = 0$, so the domain is $x \neq 3$. There is no x-intercept, because $y \neq 0$ for any value of x. The y-intercept is where $x = 0$, so $y = \frac{1}{-6}$. Thus, the y-intercept is $(0, -\frac{1}{6})$. There is a vertical asymptote where the denominator is zero (boundary of the domain). Thus, the vertical asymptote is $x = 3$. Since the power of x in the denominator (linear) exceeds the power of x in the numerator (constant), there is a horizontal asymptote of $y = 0$. The graph is shown below on the left.

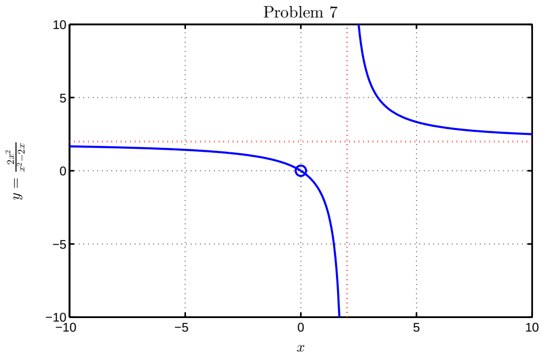




5. For $y = \frac{x+1}{x-1}$, the denominator is zero when $x - 1 = 0$, so the domain is $x \neq 1$. The x-intercept satisfies $y = 0$, which is when the numerator is zero. So $x + 1 = 0$, which gives the x-intercept $x = -1$ or $(-1, 0)$. The y-intercept is where $x = 0$, so $y = \frac{1}{-1}$. Thus, the y-intercept is $(0, -1)$. The vertical asymptote occurs where the denominator is zero (boundary of the domain). Thus, the vertical asymptote is $x = 1$. Since the power of x in the numerator and denominator is one (linear), the horizontal asymptote is given by the ratio of the coefficients, which are both 1. Thus, the horizontal asymptote is $y = 1$. The graph is shown above on the right.

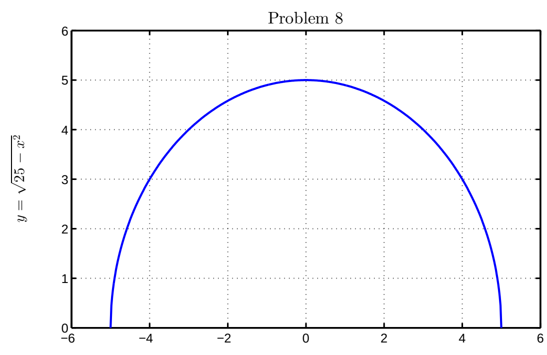
6. For $y = \sqrt{2x-4}$, the domain requires the quantity under the radical be non-negative, or $2x - 4 \geq 0$ so the domain is $x \geq 2$. The x-intercept satisfies $y = 0$ or $2x - 4 = 0$, which gives the x-intercept as $x = 2$. The y-intercept is where $x = 0$, which is outside the domain, so there is no y-intercept. The graph is shown below on the left.

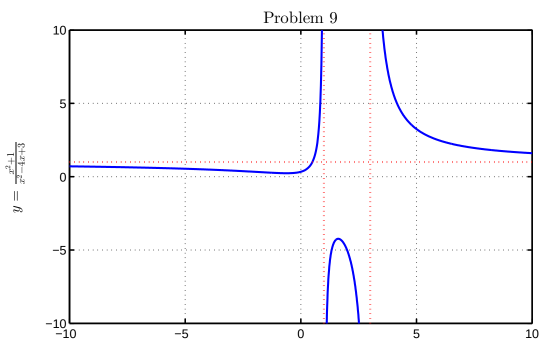




7. For $y = \frac{2x^2}{x^2-2x}$, the denominator is zero when $x^2 - 2x = 0$, so the domain is $x \neq 0,\ 2$. The x-intercept satisfies $y = 0$, which is when $2x^2 = 0$, but $x = 0$ is outside the domain, so there is no x-intercept. The y-intercept is where $x = 0$, which is outside the domain, so there is no y-intercept. The vertical asymptote occurs where the denominator is zero but the numerator is not. Thus, $x^2 - 2x = x(x-2) = 0$, gives the vertical asymptote of $x = 2$. Since the power of x in the numerator and denominator is two (quadratic), the horizontal asymptote is given by the ratio of the coefficients, which is $\frac{2}{1}$. Thus, the horizontal asymptote is $y = 2$. The graph is shown above on the right.

8. For $y = \sqrt{25 - x^2}$, the domain requires the quantity under the radical be non-negative or $25 - x^2 \geq 0$, so the domain is $-5 \leq x \leq 5$. The x-intercept satisfies $y = 0$, which is when $25 - x^2 = 0$, which gives the x-intercepts as $x = \pm 5$. The y-intercept is where $x = 0$, so $y = \sqrt{25} = 5$ so the y-intercept is $y = (0, 5)$. The graph is shown below on the left.





9. For $y = \frac{x^2+1}{x^2-4x+3}$, the denominator is zero when $x^2 - 4x + 3 = 0$, so the domain is $x \neq 1, 3$. The x-intercept satisfies $y = 0$, which is when $x^2 + 1 = 0$, which has no real solutions, so there is no x-intercept. The y-intercept is where $x = 0$, so $y = \frac{1}{3}$ so the y-intercept is $(0, \frac{1}{3})$. The vertical asymptote occurs where the denominator is zero. Thus, $x^2 - 4x + 3 = (x-3)(x-1) = 0$, gives the vertical asymptotes of $x = 1,\ 3$. Since the power of x in the numerator and denominator is two (quadratic), the horizontal asymptote is given by the ratio of the coefficients, which is 1. Thus, the horizontal asymptote is $y = 1$. The graph is shown above on the right.

10. a. Using the equation

$$V = \frac{20[S]}{10 + [S]}.$$

the horizontal asymptote is found using large $[S]$. Taking the highest powers in the numerator and denominator gives $V_{\max} = \frac{20[S]}{[S]} = 20$.

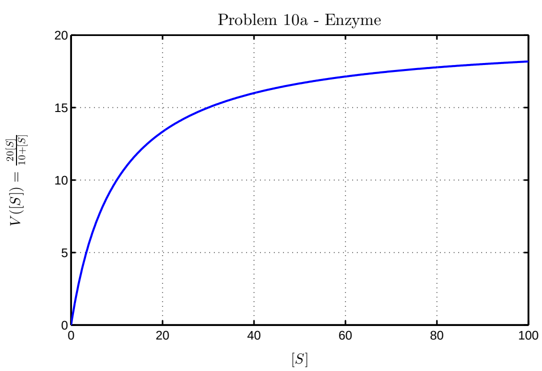
The concentration of $S([S])$ that results in the enzymatic reaction being one half its maximum velocity, $V_{\max}/2$ is

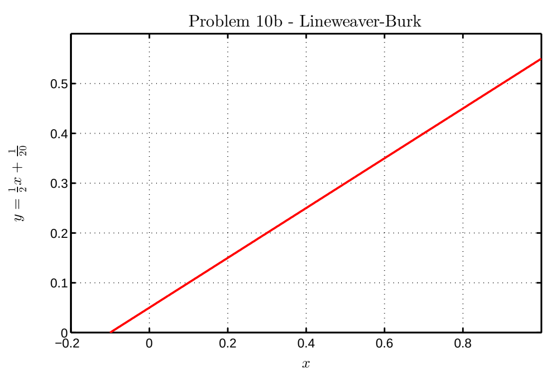
$$\begin{aligned} V_{\max}/2 = \frac{20}{2} = 10 &= \frac{20[S]}{10+[S]} \\ 10\,(10+[S]) &= 20[S] \\ 10[S] &= 100 \\ [S] &= 10 \end{aligned}$$

b. To make the substitution $x = \frac{1}{[S]}$ and $y = \frac{1}{V}$, invert the equation.

$$\begin{aligned} V &= \frac{20[S]}{10+[S]} \\ \frac{1}{V} &= \frac{10+[S]}{20[S]} \\ &= \frac{10}{20[S]} + \frac{[S]}{20[S]} \\ y &= \frac{1}{2}x + \frac{1}{20} \end{aligned}$$

The slope of this line is $m = \frac{1}{2}$. The x-intercept is where $y = 0$ so the x-intercept is $\left(-\frac{1}{10},\ 0\right)$. The y-intercept is where $x = 0$ so the y-intercept is $\left(0,\ \frac{1}{20}\right)$. The graphs are shown below.

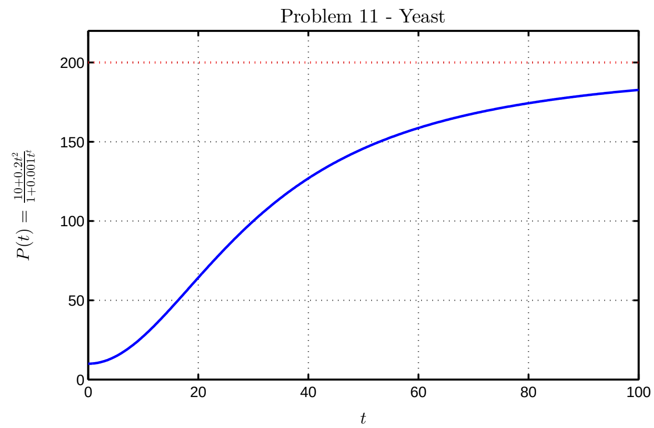




11. The density of the yeast culture at $t = 0$ satisfies $P(0) = \frac{10+0}{1+0} = 10$. Similarly at $t = 10$, $P(10) = \frac{10+0.2\cdot(10^2)}{1+0.001\cdot(10^2)} = \frac{10+20}{1+0.1} = \frac{30}{1.1} \approx 27.27$. Using the equation

$$P(t) = \frac{10+0.2t^2}{1+0.001t^2}$$

the horizontal asymptote is found using large t. Taking the highest powers in the numerator and denominator gives $P_{max} = \frac{0.2t^2}{0.001t^2} = 200$, so the horizontal asymptote is $P = 200$. The graph is shown below.



12. From the lecture notes,

$$[H^+] = \frac{1}{2}\left(-K_a + \sqrt{K_a^2 + 4K_a x}\right),$$

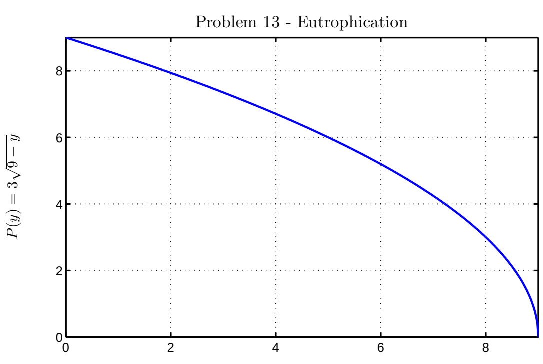
so for $K_a = 0.0001$ and a 0.1N solution

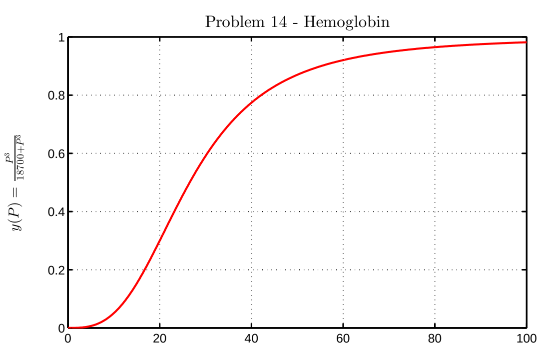
$$\begin{aligned}
[H^+] &= \frac{1}{2}\left(-0.0001 + \sqrt{0.0001^2 + 4(0.0001)(0.1)}\right) \\
&= 0.003113. \\
\text{pH} &= -\log_{10}\left([H^+]\right) = -\log_{10}(0.003113) = 2.50687
\end{aligned}$$

13. a. For $P(y) = 3\sqrt{9-y}$, the domain is found by determining where the quantity under the radical is non-negative, so $9-y \geq 0$, so the domain is $y \leq 9$. The range is from $P(0) = 3\sqrt(9) = 9$ to $P(9) = 0$. The range is therefore $0 \leq P(y) \leq 9$. The graph is shown below to the left.
b. If fish need at least 6 mmHg of dissolved O_2, then

$$\begin{aligned}
P(y) &= 3\sqrt{9-y} = 6 \\
\sqrt{9-y} &= 2 \\
9-y &= 2^2 = 4 \\
y &= 5.
\end{aligned}$$

Thus, the maximum depth for this species is $y = 5$ meters.





14. For the given function where $n = 3$ and $K = 18700$,

$$y(P) = \frac{P^3}{18700 + P^3}.$$

For 20% of hemoglobin bound, then

$$\begin{aligned}
0.2 &= \frac{P^3}{18700 + P^3} \\
0.2\left(18700 + P^3\right) &= P^3 \\
0.2(18700) &= (1 - 0.2)P^3 \\
P_{20}^3 &= \frac{0.2(18700)}{0.8} = 4675 \\
P_{20} &= \sqrt[3]{4675} = 16.721.
\end{aligned}$$

Similarly for the other percentages,

$$\begin{aligned}
P_{50}^3 &= \frac{0.5(18700)}{1 - 0.5} = 18700 \\
P_{50} &= \sqrt[3]{18700} = 26.543 \\
P_{75}^3 &= \frac{0.75(18700)}{1 - 0.75} = 56100 \\
P_{75} &= \sqrt[3]{56100} = 38.281 \\
P_{90}^3 &= \frac{0.90(18700)}{1 - 0.90} = 168300 \\
P_{90} &= \sqrt[3]{168300} = 55.211
\end{aligned}$$

The y-intercept is when $P = 0$, so the y-intercept is (0,0). The P-intercept is when $y = 0$, so the P-intercept is also (0,0) The horizontal asymptote is found using large P. Taking the highest powers in the numerator and denominator gives $y = \frac{P^3}{P^3} = 1$, so there is a horizontal asymptote at $y = 1$. The graph is shown above to the right.