Solve the following equations for x.

1. a. $x^3 + 3x^2 - 4x = 0$ b. $12x^2 - 4x^3 - x^4 = 0$

2. a. $x - \dfrac{24}{x+2} = 3$ b. $x - \dfrac{9}{x} = 0$

3. a. $\dfrac{4}{x^2} - \dfrac{3}{x} = 1$ b. $x + \dfrac{6}{x-1} = -5$

For each of the following functions, determine the domain. Find any x or y-intercepts and locate any vertical or horizontal asymptotes. Sketch the graphs of the functions.

4. $y = \dfrac{1}{2x-6}$ 5. $y = \dfrac{x+1}{x-1}$ 6. $y = \sqrt{2x-4}$

7. $y = \dfrac{2x^2}{x^2-2x}$ 8. $y = \sqrt{25-x^2}$ 9. $y = \dfrac{x^2+1}{x^2-4x+3}$

10. Suppose an enzyme satisfies the equation

$$V = \frac{20[S]}{10 + [S]}.$$

a. Find the horizontal asymptote for $V([S])$. This is the maximum velocity of the enzymatic reaction, $V_{\max}$. Find the concentration of $S([S])$ that results in the enzymatic reaction being one half its maximum velocity, $V_{\max}/2$.

b. By making the substitution $x = \frac{1}{[S]}$ and $y = \frac{1}{V}$, the enzyme reaction equation above forms a line (Lineweaver-Burk) of the form $y = mx+b$. Find the intercepts on the x and y axes. What is the slope for this line? You should sketch the graphs of the functions for $0 \leq [S] \leq 100$.

11. The growth of a culture of yeast is empirically shown to satisfy the equation

$$P(t) = \frac{10 + 0.2t^2}{1 + 0.001t^2},$$

where t is in hours and P is the density of the population (number of yeast/cc). Find the density of the yeast culture at $t = 0$ and 10 hours. The maximum possible density for this culture is given by the value of the horizontal asymptote. Find the horizontal asymptote. You should sketch a graph of $P(t)$ for $t \geq 0$.

12. Consider a weak acid with $K_a = 0.0001$. Find the $[H^+]$ for a 0.1N solution of this acid. Find the pH of this solution.

13. **Eutrophication** During an algae bloom, a pond becomes eutrophic with oxygen levels dropping to near zero at the bottom because of decaying organic matter. Suppose that the

level of O_2 dissolved in the water varies with the depth from the surface according to the function:

$$P(y) = 3\sqrt{9 - y},$$

where y is the depth in meters from the surface and P is in mmHg of dissolved O_2.

a. Assuming that the depth is nonnegative, find the domain for y and range for $P(y)$. You should sketch a graph of this function.

b. If fish need at least 6 mmHg of dissolved O_2, then how deep can the fish survive?

14. Hemoglobin binds cooperatively with O_2 in a very efficient manner. If we define y as the fraction of hemoglobin saturated with O_2 and P is the partial pressure of O_2 measured in torrs, then the relation between y and P is given by the expression:

$$y(P) = \frac{P^n}{K + P^n},$$

where $n = 3$ and $K = 18700$. Find the partial pressure of O_2 required to have 20 percent of the hemoglobin bound ($y = 0.2$). Find the partial pressures of O_2 required to have 50, 75 and 90 percent of the hemoglobin bound. Find the y and P intercepts. Determine any horizontal asymptotes. You should make a sketch of this graph with the information that you have found above on a piece of paper.