

1. Consider the following initial value problem:

$$\frac{dy}{dt} = 0.3y, \quad y(0) = 20.$$

Solve this initial value problem. Determine the value of the solution at  $t = 1$ ,  $y(1)$ .

Use Euler's method to approximate the solution  $y(1)$  using a stepsize of  $h = 0.2$  for  $t \in [0, 1]$ . Compute the error between the actual solution and the approximate solution using Euler's method.

2. Consider the following initial value problem:

$$\frac{dy}{dt} = 10 - 0.3y, \quad y(0) = 10.$$

Solve this initial value problem. Determine the value of the solution at  $t = 1$ ,  $y(1)$ .

Use Euler's method to approximate the solution  $y(1)$  using a stepsize of  $h = 0.2$  for  $t \in [0, 1]$ . Compute the error between the actual solution and the approximate solution using Euler's method.

3. A population of animals that includes emigration satisfies the differential equation

$$\frac{dP}{dt} = 0.25P - 15, \quad P(0) = 320.$$

a. Solve this differential equation and find the value of the solution at  $t = 1$ ,  $P(1)$ .

b. Use Euler's method with  $h = 0.25$  to approximate the solution at  $t = 1$ . Find the percent error between the actual solution and this approximate solution at  $t = 1$ .

4. The body temperature of a particular animal is normally  $36^\circ\text{C}$ . Suppose this animal is hit by a car at midnight ( $t = 0$ ), and the environmental temperature is approximately  $22^\circ\text{C}$ . From Newton's Law of Cooling, the temperature of the roadkill satisfies the differential equation

$$\frac{dT}{dt} = -0.2(T - 22),$$

where  $t$  is in hours.

a. Solve this differential equation, and find the temperature of the body at 2 am, *i.e.*, find the value of the solution at  $t = 2$ .

b. Use Euler's Method with  $h = 0.5$  to approximate the temperature at  $t = 2$ . Find the percent error between the actual solution and this approximate solution at  $t = 2$ .

c. Suppose that the temperature is actually dropping about  $0.5^\circ\text{C}/\text{hr}$ , then the differential equation describing the temperature of the roadkill is

$$\frac{dT}{dt} = -0.2(T - (22 - 0.5t)).$$

Use Euler's Method with  $h = 0.5$  to approximate the temperature at  $t = 2$ .

5. a. Radioactive elements are often the products of the decay of another radioactive element. A differential equation describing this situation is given by the following:

$$\frac{dR}{dt} = -0.25R + 7e^{-0.05t}, \quad R(0) = 70,$$

where  $t$  is in years. Use Euler's method with a stepsize of  $h = 1$  to find the approximate solution at  $t = 3$ .

b. Show that the actual solution is

$$R(t) = 35e^{-0.05t} + 35e^{-0.25t}.$$

Evaluate the solution at  $t = 3$  and use this solution to find the percent error of Euler's method at  $t = 3$ .