1. a. Consider the logistic growth model $P_{n+1} = 1.5P_n - 0.0025P_n^2$ with $P_0 = 50$. The next three time periods satisfy:

$$\begin{aligned} P_1 &= 1.5(50) - 0.0025(50)^2 = 68.75, \\ P_2 &= 1.5(68.75) - 0.0025(68.75)^2 \approx 91.3, \\ P_3 &= 1.5(91.3) - 0.0025(91.3)^2 \approx 116.1. \end{aligned}$$

b. At equilibrium, $P_e = 1.5P_e - 0.0025P_e^2$ or $0 = P_e(0.5 - 0.0025P_e)$. Thus,

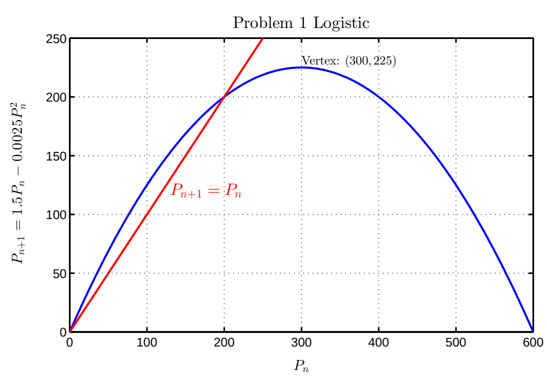
$$P_{1e} = 0 \qquad \text{or} \qquad P_{2e} = \frac{0.5}{0.0025} = 200.$$

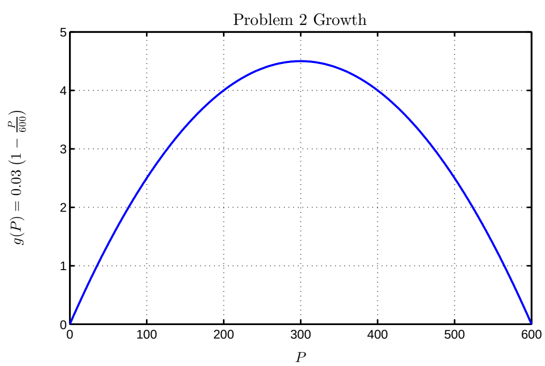
c. At the P-intercept, $F(P) = P(1.5 - 0.0025P) = 0$, so $P = 0$ or $P = \frac{1.5}{0.0025} = 600$. This is a quadratic so the vertex is halfway between these two intercepts, or $P_v = \frac{0+600}{2} = 300$ and $F(P_v) = 1.5(300) - 0.0025(300)^2 = 225$. The graph is shown below on the left

d. The derivative of the updating function, $F'(P) = 1.5 - 0.005P$. Evaluating at the equilibria gives:

$$F'(0) = 1.5 \qquad \text{or} \qquad F'(200) = 1.5 - 0.005(200) = 0.5.$$

Since $F'(0) = 1.5 > 1$, the equilibrium at $P_{1e} = 0$ is unstable (monotonically). Since $F'(200) = 0.5 < 1$, the equilibrium at $P_{2e} = 200$ is stable (monotonically).





2. a. Consider the logistic growth model $P_{n+1} = P_n + 0.03P_n\left(1 - \frac{P_n}{600}\right)$ with $P_0 = 100$. The next three time periods satisfy:

$$\begin{aligned} P_1 &= 100 + 0.03(100)\left(1 - \frac{100}{600}\right) = 102.5, \\ P_2 &= 102.5 + 0.03(102.5)\left(1 - \frac{102.5}{600}\right) = 105.05, \\ P_3 &= 105.05 + 0.03(105.05)\left(1 - \frac{105.05}{600}\right) = 107.65. \end{aligned}$$

b. The growth rate is zero when $g(P) = 0.03P\left(1 - \frac{P}{600}\right) = 0$. Thus,

$$P_{1e} = 0 \qquad \text{or} \qquad P_{2e} = 600.$$

Because this is a quadratic, the maximum growth occurs at the midpoint between the two intercepts, or $P_v = 300$ and $g(P_v) = 0.03(300)\left(1 - \frac{300}{600}\right) = 4.5$. The graph is shown above on the right.

c. At equilibrium, the growth rate is zero, so the equilibria are $P_{1e} = 0$ and $P_{2e} = 600$.

d. The updating function is $F(P) = 1.03P - \frac{0.03P^2}{600}$. The derivative of the updating function, $F'(P) = 1.03 - \frac{0.03 \cdot 2P}{600} = 1.03 - 0.0001P$. Evaluating at the equilibria gives:

$$F'(0) = 1.03 \qquad \text{or} \qquad F'(600) = 1.03 - 0.0001(600) = 0.97.$$

Since $F'(0) = 1.03 > 1$, the equilibrium at $P_{1e} = 0$ is unstable (monotonically). Since $F'(600) = 0.97 < 1$, the equilibrium at $P_{2e} = 600$ is stable (monotonically).

3. a. With the Malthusian growth model, $P_{n+1} = (1+r)P_n$ and $P_0 = 16$, the solution satisfies $P_n = 16(1+r)^n$. Since $P_2 = 16(1+r)^2 = 21$, we have

$$1 + r = \sqrt{\frac{21}{16}} \qquad \text{or} \qquad r = \sqrt{\frac{21}{16}} - 1 \approx 0.14564.$$

The population is double when $P_n = 16(1+r)^n = 32$, so $n\ln(1+r) = \ln(2)$. It follows that $n = \frac{\ln(2)}{\ln(1.14564)} = 5.0979$ hr.

b. The logistic growth model is $P_{n+1} = F(P_n) = 1.1456P_n - 0.0005P_n^2$ with $P_0 = 16$. The next three hours satisfy

$$\begin{aligned} P_1 &= 1.14564(16) - 0.0005(16)^2 = 18.202 \\ P_2 &= 1.14564(18.2024) - 0.0005(18.2024)^2 = 20.688 \\ P_3 &= 1.14564(20.6877) - 0.0005(20.6877)^2 = 23.487. \end{aligned}$$

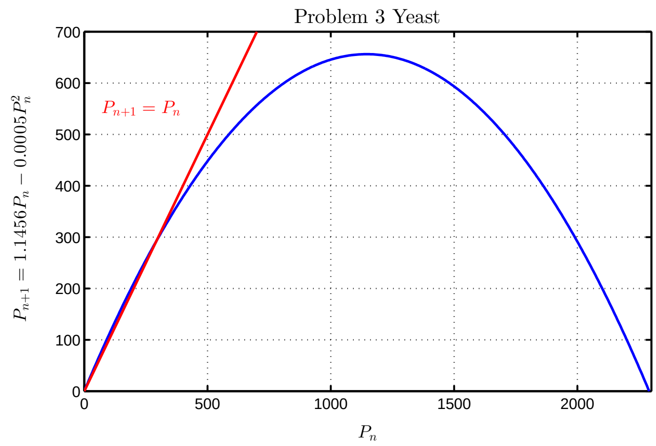
The equilibria satisfy $P_e = 1.14564P_e - 0.0005P_e^2$, so $P_e(0.14564 - 0.0005P_e) = 0$. Thus, $P_{1e} = 0$ (extinction) and $P_{2e} = \frac{0.14564}{0.0005} = 291.28$ (carrying capacity).

c. The P-intercepts satisfy $F(P) = P(1.14564 - 0.0005P) = 0$, so $P_{1i} = 0$ and $P_{2i} = \frac{1.14564}{0.0005} = 2291.28$. The vertex is midway between the two intercepts, so $P_v = \frac{2291.28+0}{2} = 1145.64$ and $F(P_v) = 1.14564(1145.64) - 0.0005(1145.64)^2 = 656.25$. The graph is shown below.

d. The derivative of the updating function is

$$F'(P) = 1.1456 - 0.001P.$$

Since $F'(0) = 1.1456 > 1$, the equilibrium at $P_{1e} = 0$ is unstable (monotonically). Since $F'(291.28) = 0.85432 < 1$, the equilibrium at $P_{2e} = 291.28$ is stable (monotonically).



4. a. With the Malthusian growth model, $P_{n+1} = (1+r)P_n$ and $P_0 = 41.8$, the solution satisfies $P_n = 41.8(1+r)^n$. Since $P_2 = 41.8(1+r)^2 = 50.7$, we have

$$1 + r = \sqrt{\frac{50.7}{41.8}} \qquad \text{or} \qquad r = \sqrt{\frac{50.7}{41.8}} - 1 \approx 0.101326.$$

The population is double when $P_n = 41.8(1+r)^n = 83.6$, so $n\ln(1+r) = \ln(2)$. It follows that $n = \frac{\ln(2)}{\ln(1.101326)} = 7.18177$ decades or 71.818 years.

b. The year 2000 is decade 5, so $P_5 = 41.8(1.101326)^5 = 67.726$ million. The percent error is $\frac{67.726-59.4}{59.4} \times 100\% = 14.0168\%$.

c. A logistic growth model for France satisfies $P_{n+1} = F(P_n) = 1.278P_n - 0.00413P_n^2$ with $P_0 = 41.8$. In 1960, $n = 1$, and in 1970 $n = 2$, so

$$\begin{aligned} P_1 &= 1.278(41.8) - 0.00413(41.8)^2 = 46.2043 \\ P_2 &= 1.278(46.2043) - 0.00413(46.2043)^2 = 50.2322 \end{aligned}$$

d. The equilibria satisfy $P_e = 1.278P_e - 0.00413P_e^2$ or $P_e(0.278 - 0.00413P_e) = 0$. It follows that $P_{1e} = 0$ or $P_{2e} = \frac{0.278}{0.00413} = 67.312$ million. The derivative of the updating function is $F'(P) = 1.278 - 0.00826P$. Since $F'(0) = 1.278 > 1$, the equilibrium at $P_{1e} = 0$ is unstable (monotonically). Since $F'(67.312) = 0.722 < 1$, the equilibrium at $P_{2e} = 67.312$ is stable (monotonically).

5. a. The Ricker's model is given by $P_{n+1} = R(P_n) = 12.2P_ne^{-0.0005P_n}$ with $P_0 = 300$. The next 3 years predicted by this model are:

$$\begin{aligned} P_1 &= 12.2(300)e^{-0.0005(300)} = 3150.191 \\ P_2 &= 12.2(3150.191)e^{-0.0005(3150.191)} = 7955.023 \\ P_3 &= 12.2(7955.023)e^{-0.0005(7955.023)} = 1817.984 \end{aligned}$$

b. The equilibria satisfy $P_e = 12.2P_ee^{-0.0005P_e}$, so either $P_{1e} = 0$ (extinction) or $12.2e^{-0.0005P_e} = 1$. This second case is equivalent to $e^{0.0005P_e} = 12.2$ or $P_{2e} = \frac{\ln(12.2)}{0.0005} = 5002.87$.

c. The updating function for this model is $R(P) = 12.2Pe^{-0.0005P}$. The derivative of the updating function,

$$R'(P) = 12.2\left(P(-0.0005)e^{-0.0005P} + e^{-0.0005P}\right) = 12.2(1 - 0.0005P)e^{-0.0005P}.$$

The P-intercept occurs where $R(P) = 0$, which occurs only when $P = 0$. The maximum of the updating function occurs where $R'(P) = 0$, so

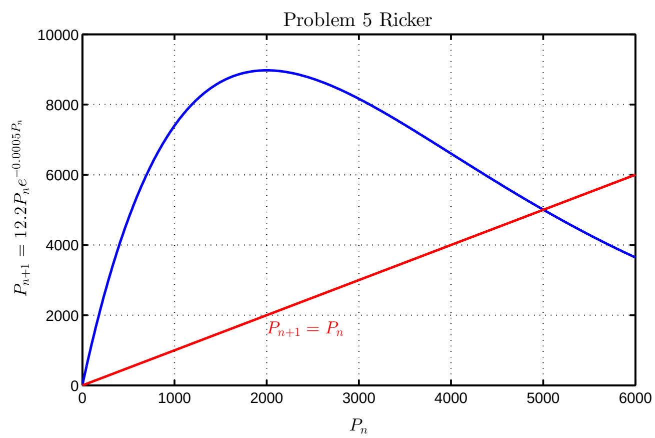
$$1 - 0.0005P = 0 \qquad \text{or} \qquad P_{max} = 2000.$$

Thus, $R(P_{max}) = 12.2(2000)e^{-0.0005(2000)} = 8976.26$. As $P \to \infty$, the exponential function dominates and tends to zero, so

$$\lim_{P\to+\infty} R(P) = 0.$$

Thus, there is a horizontal asymptote at $R = 0$. The graph of the updating function appears below.

d. We evaluate the derivative of the updating function at the equilibria. Since $R'(0) = 12.2 > 1$, the equilibrium at $P_{1e} = 0$ is unstable (monotonically). Since $R'(5002.87) = -1.5014 < -1$, the equilibrium at $P_{2e} = 5002.87$ is unstable (oscillatory).



6. a. A modification of Ricker's model that includes fishing is given by the model:

$$P_{n+1} = R(P_n) = 4P_ne^{-0.004P_n} - hP_n.$$

With $P_0 = 450$ and $h = 0.5$, we have the following next three iterations of the model:

$$\begin{aligned} P_1 &= 4(450)e^{-0.004(450)} - 0.5(450) = 72.538 \\ P_2 &= 4(72.538)e^{-0.004(72.538)} - 0.5(72.538) = 180.808 \\ P_3 &= 4(180.808)e^{-0.004(180.808)} - 0.5(180.808) = 260.495 \end{aligned}$$

b. With $h = 0.5$, the equilibria for this Ricker's model satisfy $P_e = 4P_ee^{-0.004P_e} - 0.5P_e$ or $P_e(4e^{-0.004P_e} - 1.5) = 0$. It follows that either $P_{1e} = 0$ (extinction) or $4e^{-0.004P_e} = 1.5$. The latter gives the second equilibrium, $P_{2e} = \frac{\ln(\frac{4}{1.5})}{0.004} = 245.207$. With the updating function, $R(P) = 4Pe^{-0.004P} - 0.5P$, the derivative is

$$R'(P) = 4(P(-0.004)e^{-0.004P} + e^{-0.004P}) - 0.5 = 4(1 - 0.004P)e^{-0.004P} - 0.5.$$

We evaluate the derivative at each of the equilibria. Since $R'(0) = 4 - 0.5 = 3.5 > 1$, the equilibrium at $P_{1e} = 0$ is unstable (monotonically).
Since $-1 < R'(245.207) = 4(1 - 0.004(245.207))e^{-0.004(245.207)} - 0.5 = -0.4712 < 0$, the equilibrium at $P_{2e} = 245.207$ is stable (oscillatory).

c. The non-zero equilibrium with $h = 1.7$ satisfies $P_e = 4P_e e^{-0.004P_e} - 1.7P_e$ or $1 = 4e^{-0.004P_e} - 1.7$. Thus, $e^{0.004P_e} = \frac{4}{2.7}$ or $P_{2e} = \frac{\ln(\frac{4}{2.7})}{0.004} \approx 98.261$. The derivative satisfies $R'(P) = 4(1 - 0.004P)e^{-0.004P} - 1.7$. We compute the derivative at $P_{2e} = 98.261$ and obtain $-1 < R'(98.261) = 4(1 - 0.004(98.261))e^{-0.004(98.261)} - 1.7 = -0.06121 < 0$, so this equilibrium is stable (oscillatory).

d. To find the level of fishing that drives the population to extinction, we find the equilibria for general h, so $P_e = 4P_e e^{-0.004P_e} - hP_e$. In particular, we want the second equilibrium to also be zero, so $1 = 4e^{-0.004P_e} - h$ has $P_e = 0$. It follows that $1 = 4e^{-0.004(0)} - h$ or $h = 4 - 1 = 3$. Thus, if the level of fishing h reaches 3, then the population will be driven to extinction.

7. a. Consider Hassell's model given by $P_{n+1} = H(P_n) = \frac{25P_n}{(1+0.001P_n)^2}$. If the initial population is $P_0 = 40$, then the next three years are

$$\begin{aligned}
P_1 &= \frac{25(40)}{(1+0.001(40))^2} = 924.56 \\
P_2 &= \frac{25(924.56)}{(1+0.001(924.56))^2} = 6240.4 \\
P_3 &= \frac{25(6240.4)}{(1+0.001(6240.4))^2} = 2975.96
\end{aligned}$$

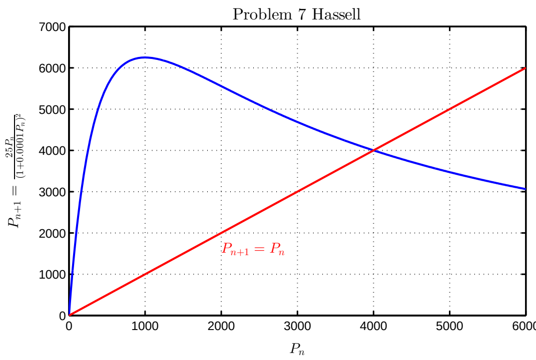
b. The equilibria satisfy $P_e = \frac{25P_e}{(1+0.001P_e)^2}$, so either $P_{1e} = 0$ (extinction) or $25 = (1+0.001P_e)^2$. The positive equilibrium solves $1 + 0.001P_{2e} = 5$ or $P_{2e} = 4000$.

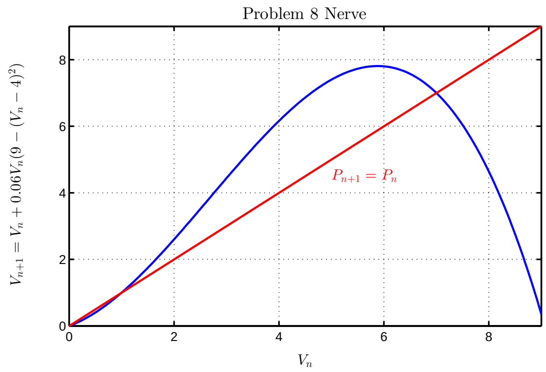
c. The updating function for this model is $H(P) = \frac{25P}{(1+0.001P)^2}$. The derivative of the updating function satisfies:

$$H'(P) = 25\frac{(1+0.001P)^2 - 2(0.001)(1+0.001P)(P)}{(1+0.001P)^4} = \frac{25(1-0.001P)}{(1+0.001P)^3}$$

The P-intercept occurs where $H(P) = 0$ so $P = 0$. The maximum of the updating function occurs where $H'(P) = 0$, which is when the numerator is zero or $1 - 0.001P = 0$. Thus, $P_{max} = 1000$ with $H(P_{max}) = \frac{25(1000)}{(1+0.001(1000))^2} = 6250$. Since the power of the denominator exceeds the power of the numerator, there is a horizontal asymptote at $H = 0$. The graph appears below to the left.

d. Since $H'(0) = 25 > 1$, the equilibrium at $P_{1e} = 0$ is unstable (monotonically). Since $-1 < H'(4000) = \frac{25(1-0.001(4000))}{(1+0.001(4000))^3} = -0.6 < 0$, the equilibrium at $P_{2e} = 4000$ is stable (oscillatory).





8. a. Consider the nerve cell model with $V_{n+1} = N(V_n) = V_n + 0.06V_n(9 - (V_n - 4)^2)$. If we consider an initial potential of $V_0 = 5$, then the next 3 time periods give:

$$\begin{aligned} V_1 &= 5 + 0.06(5)(9 - ((5) - 4)^2) \;=\; 7.4 \\ V_2 &= 7.4 + 0.06(7.4)(9 - ((7.4) - 4)^2) \;=\; 6.26336 \\ V_3 &= 6.26336 + 0.06(6.26336)(9 - ((6.26336) - 4)^2) \;=\; 7.7204. \end{aligned}$$

b. The equilibria satisfy $V_e = V_e + 0.06V_e(9 - (V_e - 4)^2)$, so $0.06V_e(9 - (V_e - 4)^2) = 0$. Thus, $V_{1e} = 0$ or $(9 - (V_e - 4)^2) = 0$. Solving the last equation gives $V_e - 4 = \pm 3$ or $V_{2e} = 1$ and $V_{3e} = 7$.

c. Since the updating function is $N(V) = V + 0.06V(9 - (V - 4)^2)$, the derivative satisfies

$$N'(V) = 1 + 0.06\left(-2V(V-4) + (9 - (V-4)^2)\right) = 1 + 0.06\left(-3V^2 + 16V - 7)\right)$$

The updating function and the identity map are shown above to the right.

d. We evaluate the derivative at each of the equilibria and determine the local behavior. Since $N'(0) = 0.58 < 1$, the equilibrium at $V_{1e} = 0$ is stable (monotonically). Since $N'(1) = 1 + 0.06\left(-3(1)^2 + 16(1) - 7)\right) = 1.36$, the equilibrium at $V_{2e} = 1$ is unstable (monotonic). Since $N'(7) = 1 + 0.06\left(-3(7)^2 + 16(7) - 7)\right) = -1.52$, the equilibrium at $V_{3e} = 7$ is unstable (oscillatory). The dynamical model for a nerve cell shows that a small stimulus ($V0 < 1$) will return to rest with $V_e = 0$. When the stimulus is larger ($V0 > 1$), then the nerve cell will fire continuously in an oscillatory manner with the voltage going above and below the unstable active equilibrium $V_e = 7$.

9. a. Consider the Allee model satisfying $N_{n+1} = A(N_n) = N_n + 0.2N_n\left(1 - \frac{1}{16}(N_n - 6)^2\right)$. If we start with an initial population of $N_0 = 4$, then the next two generations are

$$\begin{aligned} N_1 &= 4 + 0.2(4)\left(1 - \frac{1}{16}((4) - 6)^2\right) \;=\; 4.6 \\ N_2 &= 4.6 + 0.2(4.6)\left(1 - \frac{1}{16}((4.6) - 6)^2\right) \;=\; 5.4. \end{aligned}$$

b. The equilibria for this model satisfy $N_e = N_e + 0.2N_e\left(1 - \frac{1}{16}(N_e - 6)^2\right)$ or $N_e\left(1 - \frac{1}{16}(N_e - 6)^2\right) = 0$. Thus, $N_{1e} = 0$ or $\left(1 - \frac{1}{16}(N_e - 6)^2\right) = 0$. Solving the last equation gives $N_e - 6 = \pm 4$ or $N_{2e} = 2$ and $N_{3e} = 10$.

c. The derivative of the updating function

$$A(N) = N + 0.2N\left(1 - \frac{1}{16}(N-6)^2\right) = N + 0.2N\left(1 - \frac{N^2 - 12N + 36}{16}\right)$$

satisfies:

$$A'(N) = 1 - 0.2N\left(\frac{2N-12}{16}\right) + 0.2\left(1 - \frac{N^2 - 12N + 36}{16}\right) = \frac{3}{4} + \frac{3N}{10} - \frac{3N^2}{80}$$

d. We evaluate the derivative at each of the equilibria and determine the local behavior. Since $A'(0) = \frac{3}{4} < 1$, the equilibrium at $N_{1e} = 0$ is stable (monotonically). Since $A'(2) = \frac{3}{4} + \frac{3(2)}{10} - \frac{3(2)^2}{80} = 1.2$, the equilibrium at $N_{2e} = 2$ is unstable (monotonic). Since $A'(10) = \frac{3}{4} + \frac{3(10)}{10} - \frac{3(10)^2}{80} = 0$, the equilibrium at $N_{3e} = 10$ is stable (monotonic).

e. This Allee effect shows that if the flamingo population falls below $N_e = 2$ (thousand), then the population will tend toward zero (extinction). If the population is above $N_e = 2$ (thousand), then the population will tend toward the carrying capacity of $N_e = 10$ (thousand).