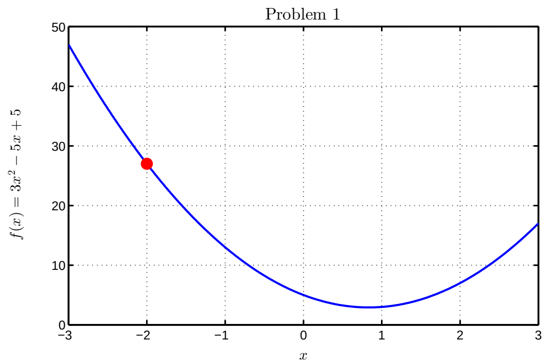
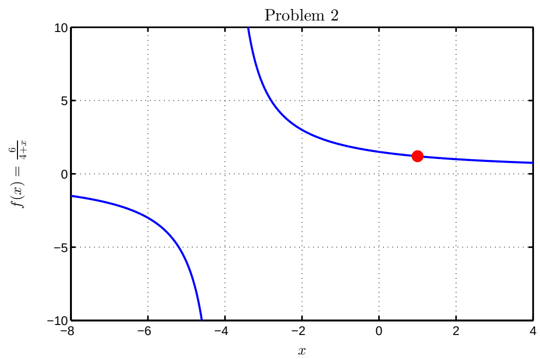
1. a. We consider the function $f(x) = 3x^2 - 5x + 5$. $f(x)$ is continuous at $x = -2$ (as are all polynomials at any value of x) with $f(-2) = 3(-2)^2 - 5(-2) + 5 = 27$. The graph is shown below to the left.

b. The limit is the same as the value for continuous functions, so

$$\lim_{x\to -2} f(x) = 27.$$





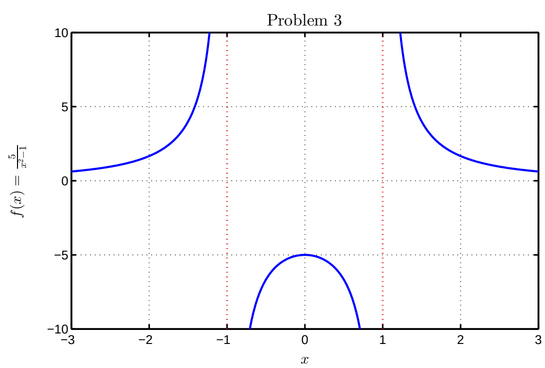
2. a. We consider the function $f(x) = \frac{6}{4+x}$. $f(x)$ is continuous at $x = 1$, as the denominator is not zero there with $f(1) = \frac{6}{4+1} = 1.2$. The graph is shown above to the right.

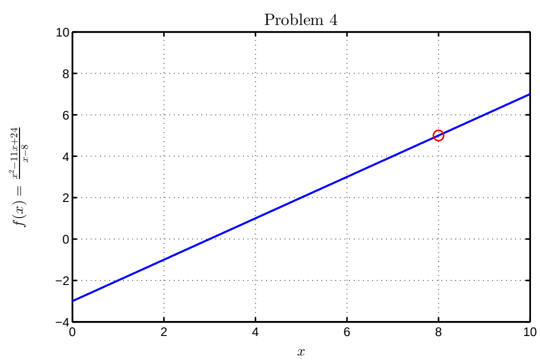
b. The limit is the same as the value for the continuous function, so

$$\lim_{x\to 1} f(x) = 1.2.$$

3. a. We consider the function $f(x) = \frac{5}{x^2-1} = \frac{5}{(x+1)(x-1)}$. There is a vertical asymptote at $x = 1$, so the function is undefined and not continuous at $x = 1$. The graph is shown below to the left.

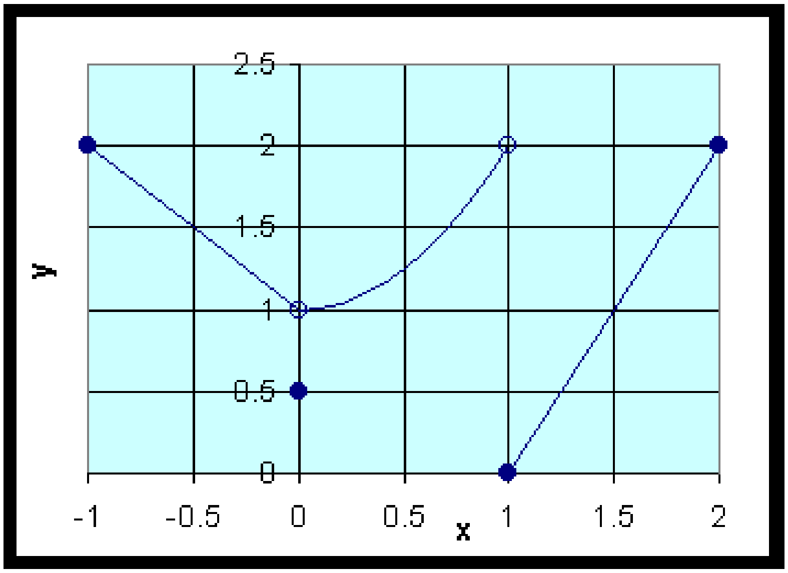
b. Since there is a discontinuity at $x = 1$, the limit doesn't exist at $x = 1$.





4. a We consider the function $f(x) = \frac{x^2-11x+24}{x-8} = \frac{(x-8)(x-3)}{x-8}$. Since the denominator is zero at $x = 8$, the function is undefined and not continuous at $x = 8$. The graph is shown above to the right.

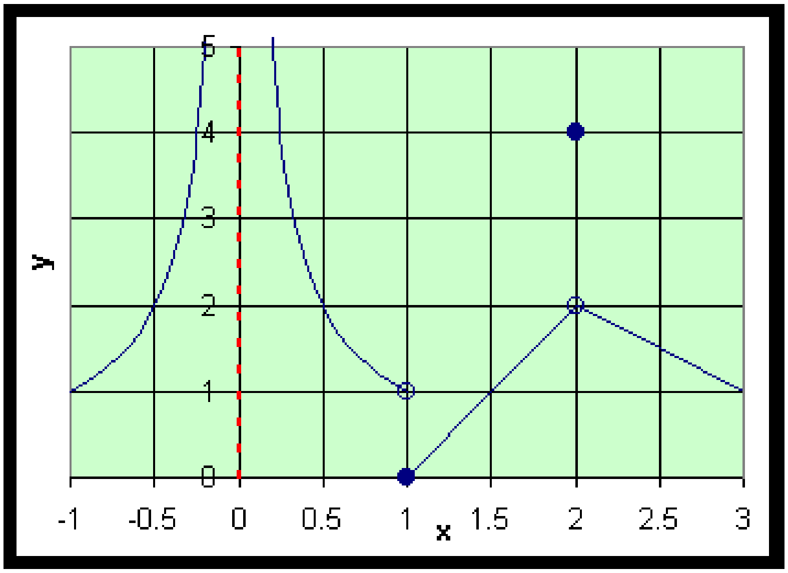
b. The function does have a limit at $x = 8$. $\lim_{x\to 8} f(x) = x - 3 = 8 - 3 = 5$.



5. Consider the function in the figure above. It is easy to see that $f(0) = 0.5$ and $f(1) = 0$ (from the solid dots). However,

$$\lim_{x\to 0} f(x) = 1,$$

as the curves defining $f(x)$ to either the right or left of $x = 0$ get arbitrarily close to 1. Near $x = 1$, there is a jump discontinuity with the function approaching 2 from the left and approaching 0 from the right. This implies that $\lim_{x\to 1} f(x)$ **does not exist**.



6. Consider the function in the figure above. It is easy to see that $f(0)$ is undefined (vertical asymptote), while $f(1) = 0$ and $f(2) = 4$ (solid dots). Since there is a vertical asymptote (discontinuity) at $x = 0$, $\lim_{x\to 0} f(x)$ **does not exist**. Similarly, there is a discontinuity at $x = 1$, so $\lim_{x\to 1} f(x)$ **does not exist**. At $= 2$, we see $f(x)$ approaching 2 from the left and similarly from the right. Thus,

$$\lim_{x\to 2} f(x) = 2.$$

7. a. We consider $f(x) = 2x - x^2$. Next we evaluate

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(2(x+h) - (x+h)^2) - (2x - x^2)}{h} \\ &= \frac{2h - 2hx - h^2}{h} = 2 - 2x - h. \end{aligned}$$

b. From the definition of the derivative, we see

$$\lim_{h\to 0} f(x) = f'(x) = 2 - 2x.$$

8. a. We consider $f(x) = \frac{3}{x+3}$. Next we evaluate

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{1}{h}\left(\frac{3}{x+h+3} - \frac{3}{x+3}\right) \\ &= \frac{1}{h}\left(\frac{3(x+3)-3(x+h+3)}{(x+h+3)(x+3)}\right) \\ &= \frac{-3h}{h(x+h+3)(x+3)} = \frac{-3}{(x+h+3)(x+3)}.\end{aligned}$$

b. From the definition of the derivative, we see

$$\lim_{h\to 0} f(x) = f'(x) = \frac{-3}{(x+3)^2}.$$