1. Consider the integral with the substitution \( u = 2x^2 - 3 \), so \( du = 4x \, dx \). It follows that
\[
\int x\sqrt{2x^2-3} \, dx = \frac{1}{4} \int (2x^2-3)^{\frac{1}{2}}(4x) \, dx = \frac{1}{4} \int u^{\frac{1}{2}} \, du \\
= \frac{1}{4} \left( \frac{2}{3} u^{\frac{3}{2}} \right) + C = \frac{1}{6} (2x^2-3)^{\frac{3}{2}} + C.
\]

2. Consider the integral with the substitution \( u = 2x - 6 \), so \( du = 2 \, dx \). It follows that
\[
\int \frac{4}{2x-6} \, dx = 2 \int \frac{2}{2x-6} \, dx = 2 \int u^{-1} \, du \\
= 2 \ln |u| + C = 2 \ln |2x-6| + C.
\]

3. Consider the integral with the substitution \( u = x^2 + 4x - 5 \), so \( du = (2x + 4) \, dx \). It follows that
\[
\int \frac{x+2}{(x^2 + 4x - 5)^3} \, dx = \frac{1}{2} \int \frac{2x+4}{(x^2 + 4x - 5)^3} \, dx = \frac{1}{2} \int u^{-3} \, du \\
= \frac{1}{2} \left( \frac{u^{-2}}{-2} \right) + C = -\frac{1}{4(x^2 + 4x - 5)^2} + C.
\]

4. Consider the integral with the substitution \( u = x^2 + 4 \), so \( du = 2x \, dx \). It follows that
\[
\int x \sin(x^2 + 4) \, dx = \frac{1}{2} \int \sin(x^2 + 4) (2x) \, dx = \frac{1}{2} \int \sin(u) \, du \\
= -\frac{1}{2} \cos(u) + C = -\frac{\cos(x^2 + 4)}{2} + C.
\]

5. Consider the integral with the substitution \( u = x^2 - 2x \), so \( du = (2x - 2) \, dx \). It follows that
\[
\int \frac{x-1}{e^{x^2-2x}} \, dx = \frac{1}{2} \int (2x-2)e^{-(x^2-2x)} \, dx = \frac{1}{2} \int e^{-u} \, du \\
= -\frac{1}{2} e^{-u} + C = -\frac{1}{2} e^{-x^2+2x} + C.
\]
6. Consider the integral with the substitution \( u = \cos(2x) + 6 \), so \( du = -2\sin(2x) \, dx \). It follows that
\[
\int \frac{5\sin(2x)}{(\cos(2x) + 6)} \, dx = \frac{5}{-2} \int \frac{-2\sin(2x)}{(\cos(2x) + 6)} \, dx = \frac{5}{2} \int \frac{1}{u} \, du = -\frac{5}{2} \ln |u| + C = -\frac{5}{2} \ln |\cos(2x) + 6| + C.
\]

7. Consider the integral with the substitution \( u = \frac{(x + 7)}{\frac{1}{2}} \), so \( du = \frac{1}{2(x + 7)^{\frac{1}{2}}} \, dx \). It follows that
\[
\int \frac{e^{\sqrt{x+7}}}{\sqrt{x+7}} \, dx = 2 \int \frac{e^\left(\frac{x+7}{\frac{1}{2}}\right)}{2(x + 7)^{\frac{1}{2}}} \, dx = 2 \int e^u \, du = 2e^u + C = 2e^{\sqrt{x+7}} + C.
\]

8. Consider the integral with the substitution \( u = 1 - x^4 \), so \( du = -4x^3 \, dx \). It follows that
\[
\int \frac{x^3}{\sqrt{1-x^4}} \, dx = -\frac{1}{4} \int (1-x^4)^{-\frac{1}{2}}(-4x^3) \, dx = -\frac{1}{4} \int u^{-\frac{1}{2}} \, du = -\frac{2}{4} u^{\frac{1}{2}} + C = -\frac{1}{2}(1-x^4)^{\frac{1}{2}} + C.
\]

9. This differential equation has only a function of \( t \) on the right, so it can be solved by simply integrating the right hand side.
\[
y(t) = \int \left( t \sqrt{t^2 + 1} \right) \, dt = \frac{1}{2} \int (t^2 + 1)^{\frac{1}{2}} 2t \, dt.
\]
With the substitution \( u = t^2 + 1 \), so \( du = 2t \, dt \),
\[
y(t) = \frac{1}{2} \int u^{\frac{1}{2}} \, du = \left( \frac{1}{2} \right) \left( \frac{2}{3} \right) u^{\frac{3}{2}} + C = \frac{1}{3}(t^2 + 1)^{\frac{3}{2}} + C.
\]
With the initial condition, \( y(0) = 5 = \frac{1}{3}(1)^{\frac{3}{2}} + C \) or \( C = \frac{14}{3} \). Thus,
\[
y(t) = \frac{1}{3}(t^2 + 1)^{\frac{3}{2}} + \frac{14}{3}.
\]

10. This differential equation has only a function of \( t \) on the right, so it can be solved by simply integrating the right hand side.
\[
y(t) = \int \left( t \sin(t^2 - 4) \right) \, dt = \frac{1}{2} \int \left( \sin(t^2 - 4) \right) 2t \, dt.
\]
With the substitution $u = t^2 - 4$, so $du = 2t \, dt$,

$$y(t) = \frac{1}{2} \int (\sin(u)) \, du = -\frac{1}{2} \cos(u) + C = -\frac{1}{2} \cos(t^2 - 4) + C.$$  

With the initial condition, $y(2) = 3 = -\frac{1}{2} \cos(2^2 - 4) + C$ or $C = \frac{7}{2}$. Thus,

$$y(t) = \frac{7}{2} - \frac{1}{2} \cos(t^2 - 4).$$

11. The differential equation, $\frac{dy}{dt} = \frac{ty}{\sqrt{t^2 - 1}}$, is a separable differential equation. Thus,

$$\int \frac{dy}{y} = \int \frac{t}{\sqrt{t^2 - 1}} \, dt = \frac{1}{2} \int \frac{2t}{\sqrt{t^2 - 1}} \, dt.$$  

With the substitution $u = t^2 - 1$, so $du = 2t \, dt$,

$$\ln |y| = \frac{1}{2} \int u^{-\frac{1}{2}} \, du = u^{\frac{1}{2}} + C = \sqrt{(t^2 - 1)} + C.$$  

Solving for $y$ gives

$$y(t) = Ae^{\sqrt{(t^2 - 1)}}, \quad \text{with} \quad A = e^C.$$  

With the initial condition, $y(1) = 4 = A$. Thus,

$$y(t) = 4e^{\sqrt{t^2 - 1}}.$$

12. The differential equation, $\frac{dy}{dt} = 0.1t(4 - y) = -0.1t(y - 4)$, is a separable differential equation. Thus,

$$\int \frac{dy}{y - 4} = -0.1 \int t \, dt.$$  

With the substitution $u = y - 4$, so $du = dy$,

$$\int \frac{du}{u} = \ln |u| = \ln |y(t) - 4| = -\frac{0.1t^2}{2} + C.$$  

Solving for $y$ gives

$$|y(t) - 4| = Ae^{-0.05t^2}, \quad \text{with} \quad A = e^C.$$  

With the initial condition, $y(0) = 10$, so $|10 - 4| = 6 = A$. Thus,

$$y(t) = 4 + 6e^{-0.05t^2}.$$  

13. The differential equation, $t \frac{dy}{dt} = 2 (\ln(t))^4$, when divided by $t$ has only a function of $t$ on the right hand side. Thus,

$$y(t) = 2 \int (\ln(t))^4 \frac{1}{t} \, dt.$$
With the substitution \( u = \ln(t) \), so \( du = \frac{dt}{t} \),

\[
y(t) = 2 \int u^4 du = 2 \frac{u^5}{5} + C = \frac{2}{5}(\ln(t))^5 + C.
\]

With the initial condition, \( y(1) = 3 = C \), so

\[
y(t) = \frac{2}{5}(\ln(t))^5 + 3.
\]

14. The differential equation, \( \frac{dP}{dt} = 0.2P \left(1 - \frac{P}{50,000}\right) \), is a logistic differential equation, which can be separated and solved following the technique shown in the lecture. This begins with the fractional decomposition:

\[
\frac{1}{P \left(\frac{P}{50,000} - 1\right)} = \frac{1}{50,000} \left(\frac{1}{\frac{P}{50,000} - 1}\right) - \frac{1}{P}.
\]

Thus, separation of variables gives

\[
\frac{1}{50,000} \int \left(\frac{1}{\frac{P}{50,000} - 1}\right) dP - \int \frac{dP}{P} = -0.2 \int dt = -0.2t + C.
\]

With the substitution, \( u = \frac{P}{50,000} - 1, \) so \( du = \frac{dP}{50,000}. \) If follows that:

\[
\int \frac{du}{u} - \ln |P(t)| = \ln \left| \frac{P(t)}{50,000} - 1 \right| - \ln |P(t)| = -0.2t + C.
\]

By properties of logarithms,

\[
\ln \left| \frac{P(t)}{50,000} - 1 \right| = -0.2t + C \quad \text{or} \quad \left| \frac{1}{50,000} - \frac{1}{P(t)} \right| = Ae^{-0.2t}.
\]

From the initial condition, \( P(0) = 1000, \) so

\[
\left| \frac{1}{50,000} - \frac{1}{1000} \right| = A = \frac{49}{50,000}.
\]

Since \( P(t) < 50,000, \) we have

\[
\frac{1}{P(t)} - \frac{1}{50,000} = \frac{49}{50,000}e^{-0.2t} \quad \text{or} \quad \frac{1}{P(t)} = \frac{1 + 49e^{-0.2t}}{50,000},
\]

so

\[
P(t) = \frac{50,000}{1 + 49e^{-0.2t}}.
\]

When the population doubles \( P(t) = 2000, \) so

\[
2000 = \frac{50,000}{1 + 49e^{-0.2t}} \quad \text{or} \quad 1 + 49e^{-0.2t} = \frac{50,000}{2000} = 25.
\]
It follows that $e^{0.2t} = \frac{49}{24}$ or $t = 5 \ln \left( \frac{49}{24} \right) \approx 3.5688 \text{ hr.}$

Since the carrying capacity is 50,000, half the carrying capacity is reached when $P(t) = 25,000 = \frac{50,000}{1 + 49e^{-0.2t}}$ or $1 + 49e^{-0.2t} = 2$. Thus, $t = 5 \ln(49) \approx 19.459 \text{ hr.}$

15. a. Consider $k(t) = \frac{0.12t}{t^2 + 1}$, so we differentiate and obtain:

$$k'(t) = 0.12 \frac{(t^2 + 1) \cdot 1 - t(2t)}{(t^2 + 1)^2} = 0.12 \frac{1 - t^2}{(t^2 + 1)^2}$$

This function has critical points at $t_{cr} = \pm 1$. It is easy to see there is a maximum at $t_{\text{max}} = 1$. The maximum value is $k(t_{\text{max}}) = 0.06$. The graph of $k(t)$ is shown below.

![Graph of k(t)]

b. The differential equation, $\frac{dV}{dt} = \frac{0.12t}{t^2 + 1} V^{\frac{2}{3}}$, is separable. It can be written in the form:

$$\int V^{-\frac{2}{3}} dV = 0.12 \int (t^2 + 1)^{-1} t \ dt = 0.06 \int (t^2 + 1)^{-1} 2t \ dt.$$ 

We make the substitution $u = t^2 + 1$, so $du = 2t \ dt$, so

$$3V^{\frac{1}{3}} = 0.06 \int u^{-1} du = 0.06 \ln |u| + C = 0.06 \ln(t^2 + 1) + C.$$ 

Solving for $V(t)$,

$$V(t) = \left( 0.02 \ln(t^2 + 1) + C/3 \right)^3.$$ 

The initial condition is $V(0) = 1$, so $C = 3$, which gives the solution

$$V(t) = \left( 1 + 0.02 \ln(t^2 + 1) \right)^3.$$ 

15. Growth rate $k(t)$

![Graph of k(t)]

c. For the cell to double in volume, $V(t) = 2 = (1 + 0.02 \ln(t^2 + 1))^3$. Thus, $\sqrt[3]{2} = 1 + 0.02 \ln(t^2 + 1)$ or $\ln(t^2 + 1) = 50 \left( \sqrt[3]{2} - 1 \right)$. It follows that

$$t^2 = e^{50(\sqrt[3]{2} - 1)} - 1 \approx 440669 \quad \text{or} \quad t \approx 664 \text{ min.}$$