

1. With the functions $f(t) = t^2 + 6t$ and $g(t) = t - 5$, we have:

a. When $t = 0$, then $f(0) = (0)^2 + 6(0) = 0$. For $t = 2$, $f(2) = (2)^2 + 6(2) = 16$.
When $t = -2$, then $g(-2) = (-2) - 5 = -7$. For $t = 3$, $g(3) = (3) - 5 = -2$.

b. Consider $f(g(t)) = (t - 5)^2 + 6(t - 5) = t^2 - 4t - 5$.
Similarly, $g(f(t)) = t^2 + 6t - 5$.

c. Evaluating the composite functions, $f(g(1)) = (1)^2 - 4(1) - 5 = -8$ and
 $g(f(1)) = (1)^2 + 6(1) - 5 = 2$.

2. With $f(x) = -x^2 + 3x + 5$, we have $f(0) = -(0)^2 + 3(0) + 5 = 5$,

$$f\left(-\frac{1}{4}\right) = -\left(-\frac{1}{4}\right)^2 + 3\left(-\frac{1}{4}\right) + 5 = \frac{67}{16},$$

$$f(a) = -(a)^2 + 3(a) + 5 = -a^2 + 3a + 5,$$

$$f\left(\frac{1}{d}\right) = -\left(\frac{1}{d}\right)^2 + 3\left(\frac{1}{d}\right) + 5 = -\frac{1}{d^2} + \frac{3}{d} + 5.$$

3. With $f(x) = \frac{5x+4}{x-4}$, we have $f(0) = \frac{0+4}{0-4} = -1$,

$$f\left(-\frac{1}{3}\right) = \frac{5\left(-\frac{1}{3}\right) + 4}{-\frac{1}{3} - 4} = -\frac{7}{13},$$

$$f(a) = \frac{5a+4}{a-4},$$

$$f\left(\frac{2}{d}\right) = \frac{5\left(\frac{2}{d}\right) + 4}{\frac{2}{d} - 4} = \frac{5 + 2d}{1 - 2d}.$$

4. With $f(x) = 1 - 5x^2$, we have $f(x + h) = 1 - 5(x + h)^2$,

$$f(x + h) - f(x) = 1 - 5(x + h)^2 - 1 + 5x^2 = -5h(2x + h),$$

$$\frac{f(x+h)-f(x)}{h} = -5(2x + h).$$

5. With $f(x) = \frac{5}{x+5}$, we have $f(x + h) = \frac{5}{(x + h + 5)}$,

$$f(x + h) - f(x) = \frac{5}{(x + h + 5)} - \frac{5}{x + 5} = \frac{-5h}{(x + h + 5)(x + 5)},$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-5}{(x + h + 5)(x + 5)}.$$

6. With $f(x) = \frac{4}{x^2}$, we have $f(x + h) = \frac{4}{(x + h)^2}$,

$$f(x + h) - f(x) = \frac{4}{(x + h)^2} - \frac{4}{x^2} = \frac{-4h(h + 2x)}{x^2(x + h)^2},$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-4(h+2x)}{x^2(x+h)^2}.$$

7. a. With $f(x) = 3x^2 + 3x + 4$ and $g(x) = 2x - 6$, the compositions are

$$f(g(x)) = 3(2x - 6)^2 + 3(2x - 6) + 4 = 12x^2 - 66x + 94,$$

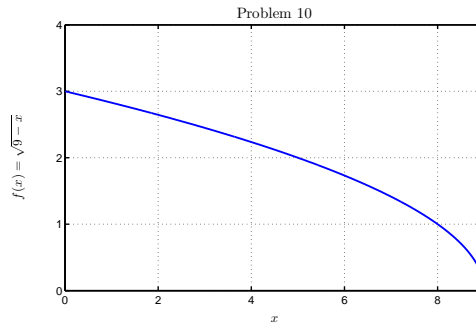
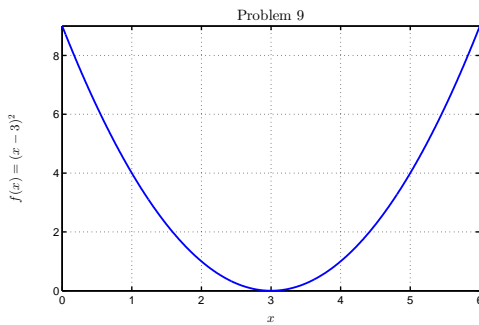
$$g(f(x)) = 2(3x^2 + 3x + 4) - 6 = 6x^2 + 6x + 2.$$

b. Evaluating the compositions at $x = 2$, we have $f(g(2)) = 12(2)^2 - 66(2) + 94 = 10$ and $g(f(2)) = 6(2)^2 + 6(2) + 2 = 38$.

8. a. The function $f(x) = 9 - x^2$ is a parabola pointing down with its vertex at $(0, 9)$. Since the maximum is at the vertex, the range of this function is $(-\infty, 9]$.

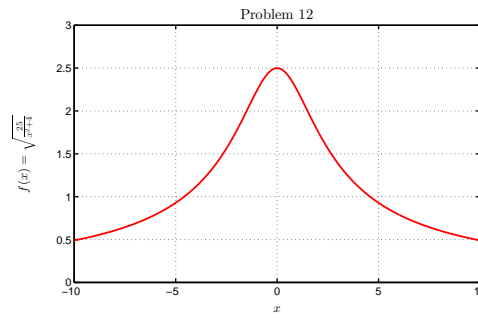
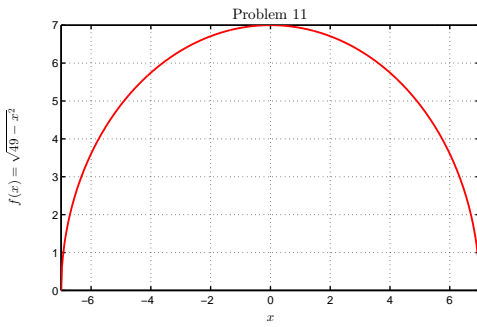
b. With the range restricted to $f(x) > 0$, we solve $9 - x^2 > 0$. This yields $x^2 < 9$ or $-3 < x < 3$, so $x \in (-3, 3)$.

9. For the function, $f(x) = (x - 3)^2$, the **domain** is all x , so $x \in (-\infty, \infty)$. The function is a parabola pointing up with a vertex at $(3, 0)$, so the **range** is $[0, \infty)$. This function has no symmetry about the origin or y -axis, so is neither even nor odd. The graph is shown below to the left.



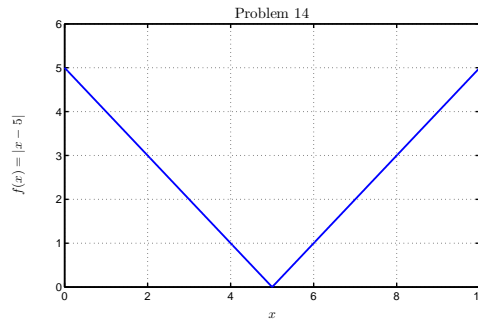
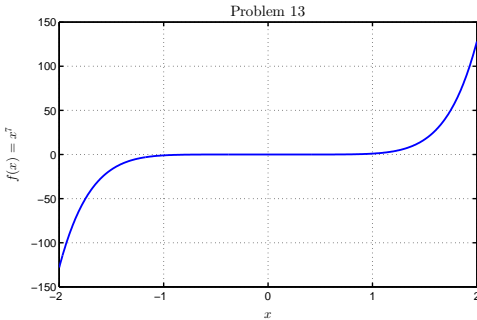
10. For $y = \sqrt{9 - x}$, the domain is determined by finding where the quantity under the radical is non-negative or $9 - x \geq 0$, so the **domain** is $x \in (-\infty, 9]$. This function is defined for positive values of y , so the **range** is $[0, \infty)$. The function has neither even nor odd symmetry. The graph is above to the right

11. For $y = \sqrt{49 - x^2}$, the domain is determined by finding where the quantity under the radical is non-negative or $49 - x^2 \geq 0$, so the **domain** is $x \in [-7, 7]$. This function creates a semi-circle. The maximum inside the domain is $y = 7$, which when combined with the function being non-negative, gives the **range** is $[0, 7]$. The graph is symmetric around the y -axis so shows **even** symmetry. The graph is below to the left.



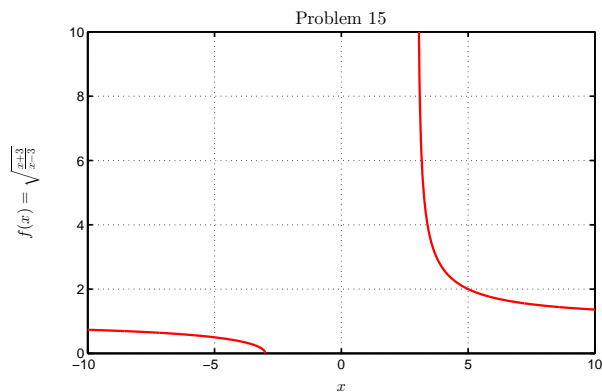
12. For $f(x) = \sqrt{\frac{25}{x^2 + 4}}$, the **domain** is all x , so $x \in (-\infty, \infty)$. The function approaches 0 for large or small x , and the maximum value is when $x = 0$ with $f(0) = \sqrt{\frac{25}{4}} = \frac{5}{2}$. It follows that the **range** is $(0, \frac{5}{2}]$. The graph is symmetric about the y -axis, so the function is **even**. The graph is above to the right.

13. For $f(x) = x^7$, the **domain** is all x , so $x \in (-\infty, \infty)$. The **range** is also $(-\infty, \infty)$. The graph is symmetric about the origin, so the function is **odd**. The graph is below to the left.



14. For $f(x) = |x - 5|$, the **domain** is all x , so $x \in (-\infty, \infty)$. The absolute value restricts the range to non-negative values, so the **range** is $[0, \infty)$. This function has no symmetry about the origin or y -axis. The graph is above to the right.

15. For $f(x) = \sqrt{\frac{x + 3}{x - 3}}$, the domain is determined by finding where the denominator is non-zero, so $x \neq 3$, and the quantity under the radical is non-negative. We solve $\frac{x + 3}{x - 3} \geq 0$, then either $x > 3$ or $x \leq -3$. It follows that the **domain** is $x \in (-\infty, -3] \cup (3, \infty)$. From the domain, we find the **range** is $[0, 1) \cup (1, \infty)$. This function has no symmetry. The graph is shown below.



16. a. From the lectures,

$$\begin{aligned}
 J(m) &= e_1^2 + e_2^2 + e_3^2 = (2 - 1m)^2 + (4.1 - 2m)^2 + (9.8 - 5m)^2 \\
 &= 4 - 4m + m^2 + 16.81 - 16.4m + 4m^2 + 96.04 - 98.0m + 25m^2 \\
 &= 30m^2 - 118.4m + 116.85.
 \end{aligned}$$

The m_v -value of the vertex is given by $m_v = \frac{118.4}{2(30)} = 1.973333$. From this, we substitute into the sum of squares function $J(m_v)$ to obtain the least sum of squares

$$J(1.973333) = 30(1.973333)^2 - 117.8(1.973333) + 115.69 = 0.028666667.$$

b. The best model is

$$A = 1.973333c,$$

so if $A = 3.5$, then the concentration of the unknown is $c = \frac{3.5}{1.973333} = 1.7736$ mM. If $A = 6.2$, then the concentration of the unknown is $c = \frac{6.2}{1.973333} = 3.1419$ mM.

17. a. From the lectures,

$$\begin{aligned}
 J(k) &= e_1^2 + e_2^2 + e_3^2 = (20 - 3.3k)^2 + (2 - 0.5k)^2 + (12 - 2k)^2 \\
 &= 400 - 132k + 10.89k^2 + 4 - 2k + 0.25k^2 + 144 - 48k + 4k^2 \\
 &= 15.14k^2 - 182k + 548.
 \end{aligned}$$

The k_v -value of the vertex is given by $k_v = \frac{182}{2(15.14)} = 6.0106$. From this, we substitute into the sum of squares function $J(k)$ to obtain the least sum of squares

$$J(6.0106) = 15.14(6.0106)^2 - 182(6.0106) + 548 = 1.038.$$

b. Using the best slope in the model, we see that her great-grandfather has a height of $d = 6.0106(1) = 6.0106$ ft, so her mother has the better memory.