

1. Consider the functions

$$f(t) = t^2 + 6t \quad \text{and} \quad g(t) = t - 5.$$

a. Evaluate $f(0)$, $f(2)$, $g(-2)$, and $g(3)$.

b. Write the composite function $f(g(t))$ in simplest form. Also, write the composite function $g(f(t))$ in simplest form.

c. Evaluate $f(g(1))$ and $g(f(1))$.

2. Consider the function

$$f(x) = -x^2 + 3x + 5.$$

Evaluate $f(0)$, $f(-\frac{1}{4})$, $f(a)$, $f(\frac{1}{d})$

3. Consider the function

$$f(x) = \frac{5x + 4}{x - 4}.$$

Evaluate $f(0)$, $f(-\frac{1}{3})$, $f(a)$, $f(\frac{2}{d})$.

For the following functions, evaluate and simplify the following expressions:

$$f(x + h), \quad f(x + h) - f(x), \quad \frac{f(x + h) - f(x)}{h}.$$

4. $f(x) = 1 - 5x^2$.

5. $f(x) = \frac{5}{x + 5}$.

6. $f(x) = \frac{4}{x^2}$

7. Consider the functions

$$f(x) = 3x^2 + 3x + 4 \quad \text{and} \quad g(x) = 2x - 6.$$

a. Write the composite function $f(g(x))$ in simplest form. Also, write the composite function $g(f(x))$ in simplest form.

b. Evaluate $f(g(2))$ and $g(f(2))$.

8. Consider the function

$$f(x) = 9 - x^2.$$

a. Find the range of this function (assuming a domain of all x).

b. Find the domain of $f(x)$, if the range of f is restricted to $f(x) > 0$.

For the following functions, find the domain and range. Write the answers in interval notation. State if the function is ODD, EVEN, or NEITHER.

$$9. f(x) = (x - 3)^2, \quad 10. y = \sqrt{9 - x}, \quad 11. y = \sqrt{49 - x^2},$$

$$12. f(x) = \sqrt{\frac{25}{x^2 + 4}}, \quad 13. f(x) = x^7, \quad 14. f(x) = |x - 5|,$$

$$15. f(x) = \sqrt{\frac{x + 3}{x - 3}}.$$

16. The Lambert-Beer law for absorbance of light by a spectrophotometer is a linear relationship, which can have the form

$$A = mc,$$

where c is the concentration of the sample, A is absorbance, and m is the slope that must be determined from experiments.

a. Below are data collected on samples from a collection of acid standards using an acid indicator.

c (mM)	1	2	5
A	2.0	4.1	9.8

The sum of squares error of the line fitting the data is given by a quadratic function of the form:

$$J(m) = b_2m^2 + b_1m + b_0.$$

Use the data from the table to find the coefficients in the quadratic function above. The vertex of this quadratic function gives the value of the best slope m_v , while the $J(m_v)$ value of the vertex gives the least sum of squares error.

b. Use this model (with the best value of $m = m_v$) to determine the concentration of an unknown acid with absorbances of $A = 3.5$ and 6.2 .

17. In looking through some old photos, a woman finds a picture of her great-grandfather standing near the family home, where she now lives. In the photograph, she measures the height of the roofline, which she knows to be 20 ft, as 3.3 cm. The 2 ft wide window measures 0.5 cm on the photo, and the distance from the front door to the oak tree at the driveway is 12 feet, which is 2 cm in the photograph.

a. The conversion of measurements in the photo p to measurements in actual distance d is given by the formula

$$d = kp.$$

The sum of squares error of the line fitting the measurements in the photo is given by a quadratic function of the form:

$$J(k) = b_2k^2 + b_1k + b_0.$$

Use the data from the description above to find the coefficients in the quadratic function above. Find the vertex of this quadratic function, which gives the value of the best slope k_v , while the $J(k_v)$ value of the vertex gives the least sum of squares error.

b. In the photograph, her great-grandfather is 1 cm tall. Her mother remembers her grandfather as a tall man of about 6 ft , whereas her father thinks he was shorter, about 5 ft 6 inches (5.5 ft). Use the model (with the best value of k) to predict the height of the great-grandfather and determine whether the mother or father better remembers the height of her great-grandfather.