1. a. Since $P_0 = 50,000$ and $r = 0.08$, the general solution to this Discrete Malthusian growth model is given by

$$P_n = 50,000(1.08)^n.$$

It follows that

$$\begin{aligned} P_1 &= 50,000(1.08) = 54,000 \\ P_2 &= 50,000(1.08)^2 = 58,320 \\ P_3 &= 50,000(1.08)^3 = 62,986. \end{aligned}$$

The amount of time required for this population to double is found by solving

$$\begin{aligned} 50,000(1.08)^n &= 100,000 \\ (1.08)^n &= 2 \\ n \cdot \ln(1.08) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.08)} = 9.0066 \text{ hr} \end{aligned}$$

b. With $P_0 = 250,000$ and $r = 0.06$, the general solution to this Discrete Malthusian growth model is given by

$$P_n = 250,000(1.06)^n.$$

It follows that

$$\begin{aligned} P_1 &= 250,000(1.06) = 265,000 \\ P_2 &= 250,000(1.06)^2 = 280,000 \\ P_3 &= 250,000(1.06)^3 = 297,754. \end{aligned}$$

The amount of time required for this population to double is found by solving

$$\begin{aligned} 250,000(1.06)n &= 500,000 \\ (1.06)n &= 2 \\ n \cdot \ln(1.06) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.06)} = 11.8957 \text{ hr.} \end{aligned}$$

2. a. Since the population of China in 1980 was 985 million and in 1990 was 1,137 million, then

$$\frac{P_1}{P_0} = \frac{1137}{985} = 1.1543 = 1 + r.$$

Thus, the growth constant is $r = 0.1543$. It follows that the general solution is given by

$$P_n = 985(1.1543)^n,$$

where n is in decades after 1980. The population in 2000 satisfies $P_2 = 985(1.1543)^2 = 1312$ million, while the population in 2050 is given by $P_7 = 985(1.1543)^7 = 2689$ million.

b. Doubling time is given by

$$\begin{aligned} 2 &= (1.1543)^n \\ n &= \frac{\ln(2)}{\ln(1.1543)} = 4.83 \text{ decades} \end{aligned}$$

so the doubling time is 48.30 years.

3. a. Since the population of the U. S. in 1980 was 227 million and in 1990 was 249 million, then

$$\frac{P_1}{P_0} = \frac{249}{227} = 1.09692 = 1 + r.$$

Thus, the growth constant is $r = 0.09692$. It follows that the general solution is given by

$$P_n = 227(1.09692)^n,$$

where n is in decades after 1980. The population in 2000 satisfies $P_2 = 227(1.09692)^2 = 273.1$ million, while the population in 2020 is given by $P_4 = 227(1.09692)^4 = 328.6$ million.

b. Since the population of Mexico in 1980 was 69 million and in 1990 was 85 million, then

$$\frac{P_1}{P_0} = \frac{85}{69} = 1.2319 = 1 + r.$$

Thus, the growth constant is $r = 0.2319$. It follows that the general solution is given by

$$P_n = 69(1.2319)^n,$$

where n is in decades after 1980. The population in 2000 satisfies $P_2 = 69(1.2319)^2 = 104.7$ million, while the population in 2020 is given by $P_4 = 69(1.2319)^4 = 158.9$ million. The amount of time required for this population to double is found by solving:

$$\begin{aligned} 69(1.2319)^n &= 138 \\ (1.2319)^n &= 2 \\ n \cdot \ln(1.2319) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.2319)} = 3.3235 \text{ decades} = 33.24 \text{ years.} \end{aligned}$$

c. The two populations are equal when

$$\begin{aligned} 227(1.09692)^n &= 69(1.2319)^n \\ \left(\frac{1.2319}{1.09692}\right)^n &= \frac{227}{69} \\ n \ln(1.12305) &= \ln(3.290) \\ n &= \frac{\ln(3.290)}{\ln(1.12305)} = 10.262 \text{ decades} = 102.62 \text{ years.} \end{aligned}$$

Thus, the population of Mexico will first exceed that of U. S. in 103 years from 1980, with Mexico having a population of 591.2 million and U. S. having a population of 588.6 million.

4. a. Since the population of the U. S. in 1880 was 50.2 million and in 1890 was 62.9 million,then

$$\begin{aligned}\frac{P_{10}}{P_0} &= \frac{62.9}{50.2} = 1.2530 = (1+r)^{10} \\ 1.25299^{\frac{1}{10}} &= 1 + r = 1.022809.\end{aligned}$$

Thus, the growth constant is $r = 0.022809$. It follows that the general solution is given by

$$P_n = 50.2(1.022809)^n,$$

where n is in years after 1880.

b. From the general formula, the population in 1900 is

$$P_{20} = 50.2(1.022809)^{20} = 78.812 \text{ million.}$$

Since the actual population is only 76.0 million, the percent error satisfies

$$100\left|\frac{78.7 - 76.0}{76.0}\right| = 3.7\ \%.$$

So the model predicts a population of 78.8 million in 1900, which has an error of 3.7% from the actual census data.

c. The amount of time required for this population to double is found by solving

$$\begin{aligned}50.2(1.0228)^n &= 100.4 \\ (1.0228)^n &= 2 \\ n \cdot \ln(1.0228) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.0228)} \\ &= 30.73 \text{ years.}\end{aligned}$$

This model predicts that the population would double in 30.73 years.

5. Below is a table of the Malthusian growth model for the U. S. after 1910, with population in millions, using the general form

$$P_n = 91.97(1 + 0.15)^n,$$

where n is the number of decades after 1910. The first column is the date, the second is the actual population. The third column is the prediction of population from the model, and the fourth column is the calculation of the percentage error of the model from the actual population.

Year	Population	Model	% Error
1910	91.97	91.97	0.00
1920	105.71	105.77	0.05
1930	122.78	121.63	-0.93
1940	131.67	139.87	6.23
1950	151.33	160.86	6.30
1960	179.32	184.98	3.16
1970	203.30	212.73	4.64
1980	226.55	244.64	7.99
1990	248.71	281.34	13.12

The population doubles where

$$\begin{aligned} 2 &= (1+0.15)^n \\ n &= \frac{\ln(2)}{\ln(1.15)} = 4.95948 \text{ decades} = 49.59 \text{ years.} \end{aligned}$$

So the year in which the population doubles is $1910 + 50 = 1960$.

6. a. The general solution for the population of herbivores satisfies

$$y_n = 2000(1.05)^n.$$

It follows that $y_1 = 2000(1.05)^1 = 2100$, and $y_3 = 2000(1.05)^3 = 2315$.

b. The general solution for the competing population of herbivores satisfies

$$y_n = 500(1.07)^n.$$

The time required for this population to double is found by solving

$$\begin{aligned} 500(1.07)^n &= 1000 \\ (1.07)^n &= 2 \\ n \cdot \ln(1.07) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.07)} = 10.24 \text{ years.} \end{aligned}$$

So this population doubles in 10.24 years. The population at year 10 is $z_{10} = 500(1.07)^{10} = 984$.

c. The two populations are equal when

$$\begin{aligned} 2000(1.05)^n &= 500(1.07)^n \\ \left(\frac{1.07}{1.05}\right)^n &= \frac{2000}{500} = 4 \\ n \cdot \ln(1.01905) &= \ln(4) \\ n &= \frac{\ln(4)}{\ln(1.01905)} = 73.47 \text{ years.} \end{aligned}$$

At this time the populations will be $500(1.07)^{73.47} = 72077$.

7. a. The general solution to this Malthusian growth problem is given by

$$P_n = 5000(1.015)^n.$$

So $P_{60} = 5000(1.015)^{60} = 12216$, the population after one hour is 12,216 bacteria. The time required for this population to double is found by solving

$$\begin{aligned} 5000(1.015)^n &= 10,000 \\ (1.015)n &= 2 \\ n \cdot \ln(1.015) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.015)} = 46.56 \text{ min.} \end{aligned}$$

It takes 46.56 minutes for this population to double.

b. If the culture starts with 1000 bacteria, then we can write the general solution,

$$B_n = 1000(1+r)^n.$$

If the population doubles in 40 min, then

$$\begin{aligned} 1000(1+r)^{40} &= 2000 \\ (1+r)^{40} &= 2 \\ 1+r &= (2)^{\frac{1}{40}} = 1.01748 \\ r &= 0.01748. \end{aligned}$$

Thus, the general solution is given by $B_n = 1000(1.01748)^n$. $B_{60} = 1000(1.01748)^{60} = 2828$. Thus this population is 2,828 bacteria after one hour. The two populations are equal if

$$\begin{aligned} 5000(1.015)^n &= 1000(1.01748)^n \\ \left(\frac{1.01748}{1.015}\right)^n &= \frac{5000}{1000} = 5 \\ n \cdot \ln(1.002443) &= \ln(5) \\ n &= \frac{\ln(5)}{\ln(1.002443)} = 659.6 \text{ min.} \end{aligned}$$

The populations will be equal in 659.6 minutes (almost 11 hours).

8. a. The Malthusian growth model for estimating Italy's population is given by

$$P_n = 50.2(1.061)^n.$$

where n is in decades from 1960. It follows that the population in 1990 is $P_3 = 50.2(1.061)^3 =$ 59.96 million, while in 2000 it is $P_4 = 50.2(1.061)^4 = 63.62$ million. The amount of time required for the population to double, based on this growth rate, is found by solving

$$\begin{aligned} 50.2(1.061)^n &= 100.4 \\ (1.061)^n &= 2 \\ n \cdot \ln(1.061) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.061)} = 11.71 \text{ decades} = 117.1 \text{ years.} \end{aligned}$$

b. The Nonautonomous Malthusian growth model satisfies

$$P_{n+1} = (1.069 - 0.018n)P_n \qquad \text{with} \qquad P_0 = 50.2.$$

The table below gives the solution to the Nonautonomous Malthusian growth model. The first column is the date. The second column is n. The third column gives the population for that date in millions. The fourth column is the growth factor $k(n)$. The new population P_{n+1} is found by multiplying the third and fourth columns, which becomes the population prediction for Italy's population in the next decade (entered in the next row third column).

Year	n	P_n	$1+k(n)$
1960	0	50.2	1.069
1970	1	53.66	1.051
1980	2	56.40	1.033
1990	3	58.26	1.015
2000	4	59.14	0.997

This model predicts the populations to be 58.26 million in 1990 and 59.14 million in 2000. The growth term

$$\begin{aligned} k(t) &= 0.069 - 0.018t = 0 \text{ when} \\ t &= \frac{0.069}{0.018} \\ &= 3.83 \text{ decades} = 38.3 \text{ years.} \end{aligned}$$

The population would level off in $1960 + 38 = 1998$.

c. The percent error in 1990 is given by (from part a.) $100\left|\frac{59.96-56.8}{56.8}\right| = 5.56\%$. (from part b.) $100\left|\frac{58.26-56.8}{56.8}\right| = 2.57\%$. The percent error in 2000 is given by (from part a.) $100\left|\frac{63.62-57.9}{57.9}\right| = 9.87\%$. (from part b.) $100\left|\frac{59.14-57.9}{57.9}\right| = 2.13\%$. The small population growth in Italy today is entirely due to immigration, which is consistent with the predicted leveling off in 1998.