

1. Let P_n be the population of some organism after n hours. Suppose that the organism satisfies the Malthusian growth model

$$P_{n+1} = (1 + r)P_n$$

with a growth rate r and an initial population P_0 .

a. Let $P_0 = 50,000$ and $r = 0.08$. Find the population of the organism at the end of each of the first 3 hours, *i.e.*, find P_1 , P_2 , and P_3 . Also, determine the amount of time required for this population to double.

b. Repeat the process in Part a for $P_0 = 250,000$ and $r = 0.06$.

2. The population of China in 1980 was about 985 million, and a census in 1990 showed that the population had grown to 1,137 million. Assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980 and P_n is the population n decades after 1980.

a. Use the data above to find the growth constant r . Predict the populations in the years 2000 and 2050 in millions.

b. How long does it take for China's population to double (in years)?

3. a. The population of the U. S. in 1980 was about 227 million, and a census in 1990 showed that the population had grown to 249 million. Assume that this population grows according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980, and P_n is the population n decades after 1980. Use the data above to find the growth constant r . Predict the populations in the years 2000 and 2020 in millions.

b. In 1980, the population of Mexico was 69 million, while in 1990, it had grown to 85 million. Assume its population is also growing according to a Malthusian growth law. Find its rate of growth per decade and predict its population in 2000 and 2020. How long does it take for Mexico's population to double?

c. If these countries continue to grow according to these Malthusian growth laws, then determine the first year when Mexico's population will exceed that of the U. S. and determine their populations at that time. (Round the date up to the nearest year, for example 2054.)

4. The population of the United States was about 50.2 million in 1880 and 62.9 million in 1890. Let 1880 be represented by P_0 and assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n$$

where n is in years.

a. Use the data above to find the **annual growth rate** r .

b. Use the model to predict the population in the year 1900. The actual population was about 76.0 million. What is the percent error between the model and the actual census data?

c. According to the model, how long until the U. S. population doubled from its 1880 level?

5. Take $r = 0.15$ and $P_0 = 91.97$ million (the population of the U. S. in 1910). Use the Malthusian growth model

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1910 and P_n is the population n decades after 1910. Simulate this model and estimate the population for 1950 and 1980. The actual population in 1950 was 151.33 million, and the actual population in 1980 was 226.55 million. Find the percent error in 1950 and in 1980 by using the model to predict the population. In what year does the population double with this model?

6. a. A population of herbivores satisfies the growth equation

$$y_{n+1} = 1.05y_n,$$

where n is in years. If the initial population is $y_0 = 2000$, then determine the populations y_1 and y_3 .

b. A competing group of herbivores satisfies the growth equation

$$z_{n+1} = 1.07z_n.$$

If the initial population is $z_0 = 500$, then determine how long in years it takes for this population to double. What is the population of these herbivores after 10 years, z_{10} ?

c. Find when the two populations are equal, and the individual population(s) at this time.

7. a. A culture of bacteria satisfies the Malthusian growth equation

$$P_{n+1} = 1.015P_n, \quad P_0 = 5000,$$

where n is in minutes. Find the population after one hour, P_{60} , and determine how long it takes for this culture to double.

b. Another culture of bacteria satisfies a similar Malthusian growth law

$$B_{n+1} = (1 + r)B_n.$$

Suppose that this culture doubles in 40 min and starts with 1000 bacteria. Find the value of r . What is the population of this bacteria after one hour, B_{60} ? Determine how long in minutes until the population of this bacteria is the same as the original culture from Part a.

8. Many European countries are leveling off and their population will soon begin to decline as couples produce on average less than two children per couple. Italy is the slowest growing country in the world. In 1960, Italy had 50.2 million people. In 1970 and 1980, Italy had 53.7 and 56.5 million people, respectively.

a. The average growth rate for the decades listed above is 6.1% per decade. Let $P_0 = 50.2$ with $r = 0.061$ and n as the number of decades after 1960. Use the Malthusian growth model,

$$P_{n+1} = (1 + r)P_n,$$

to estimate the population of Italy in 1990 and 2000. At this growth rate, how long in years would it take Italy's population to double?

b. Closer examination of the data shows that the growth rate between 1960 and 1970 is about 7.0%, while between 1970 and 1980 the growth rate is about 5.2%. These two growth rates suggest a declining growth rate of the form

$$k(n) = 0.069 - 0.018n,$$

with n being the number of decades after 1960. The Nonautonomous Malthusian Growth model is given by:

$$P_{n+1} = (1.069 - 0.018n)P_n,$$

with $P_0 = 50.2$. Use this model to estimate the population of Italy in 1990 and 2000. In which year does this model predict that Italy's population will level off and begin declining? (Hint: This is when $k(n) = 0$.)

c. Census data on Italy show that its population in 1990 was 56.8 million and in 2000, it was 57.9 million. Find the percent error for each year between the actual census data and the predictions you made in Parts a and b.