

Evaluate the following definite integrals:

1.  $\int_{-1}^3 (2 - x + x^2) dx,$

2.  $\int_0^4 (x^2 + 3 - e^{-x}) dx,$

3.  $\int_{-1}^5 \frac{dx}{\sqrt{6+2x}},$

4.  $\int_1^5 \frac{x^2 + 1}{x} dx,$

5.  $\int_0^\pi (4t + \cos(2t)) dt,$

6.  $\int_0^{\pi/2} \frac{\cos(x)}{1 + \sin(x)} dx,$

7.  $\int_3^4 \frac{2x dx}{\sqrt{25 - x^2}},$

8.  $\int_0^\pi (9t^2 - \sin(4t)) dt,$

9.  $\int_{-2}^2 \left( \frac{1}{x+3} + e^{2x} \right) dx,$

10.  $\int_0^7 \frac{4x}{(x^2 + 1)^2} dx.$

11. Find the area bounded by the function  $y = 4 - x^2$  and the  $x$ -axis. Sketch the graph of the region.

12. Find the area between the function  $y = 3 \sin(2x)$  and the  $x$ -axis for  $0 \leq x \leq \pi/2$ . Sketch the graph of the region.

13. Consider the curves  $y = x + 3$  and  $y = x^2 + x - 6$ .

a. For the line find the intercepts and slope. For the parabola find the the intercepts and vertex. Sketch the graph of these curves.

b. Find the points of intersection of these curves.

c. Find the area between the two curves.

14. Below are six years of data from some particular animal population (in thousands):

Year	0	1	2	3	4	5	6
Population	53	37	39	54	70	68	52

a. Find the average population in the usual manner.

b. These data are fitted pretty well by the function

$$P(t) = 53 - 18 \sin\left(\frac{\pi}{3}t\right).$$

Use this function to find when the maximum and minimum populations occur and what their values might be. Sketch a graph of this curve and show the data points from the table.

c. An alternate (often considered more accurate) method of computing the average population is given by the definite integral, which you should compute.

$$P_{ave} = \frac{1}{6} \int_0^6 P(t) dt.$$

15. Two researchers analyze seven years of population data for a particular animal that is given in the table below.

Year	0	1	2	3	4	5	6	7
Population	12	18	27	32	28	17	12	21

a. The first researcher fits the data with the quartic equation:

$$P(t) = \frac{1}{4}t^4 - 3t^3 + 9t^2 + 12.$$

Find  $P'(t)$ , the minimum and maximum populations and the times at which these occur, using this approximation to the data.

b. Find the average population by computing the definite integral

$$P_{ave} = \frac{1}{7} \int_0^7 P(t) dt.$$

c. The second researcher fits the data with the curve

$$Q(t) = 22 - 10 \cos\left(\frac{\pi}{3}t\right).$$

Find when and where the minimum and maximum populations occur using this function. Sketch a graph of this curve.

d. Find the average population with  $Q(t)$  by computing the definite integral

$$Q_{ave} = \frac{1}{7} \int_0^7 Q(t) dt.$$