Integrate the following:

1. $\int \left(4t^3 + 1 - e^{-5t}\right) dt,$

2. $\int \left(2t - \frac{6}{t^2} + \cos(4t)\right) dt$

1a. $\int \left(2 - \sqrt{x} + 4x^{1/3}\right) dx,$

2a. $\int \left(e^{-x} - \frac{3}{x} - x\right) dx$

3. $\int \left(2x^{-3} - \frac{2}{\sqrt{x}} + e^{4x}\right) dx,$

4. $\int \left(\frac{1}{5x} - 14\sin(7x) + 6x^2\right) dx$

5. $\int \left(6t^5 - \frac{4}{t^3} + \frac{2}{e^{2t}}\right) dt,$

6. $\int \left((t^3 - 1)^2 - \cos(2t)\right) dx$

Solve each of these initial value problems. (Note that these problems may use techniques from previous sections.)

7. $\frac{dy}{dt} = 2 - 0.1e^{-t}, \qquad y(0) = 10$

8. $\frac{dy}{dt} = \frac{2}{t^2}, \qquad y(1) = 1$

9. $\frac{dy}{dt} + 2y = 2, \qquad y(0) = 5$

9a. $\frac{dy}{dt} = 5 - \frac{y}{2}, \qquad y(0) = 15$

10. $\frac{dy}{dt} = \cos(2t) + 2t, \qquad y(0) = 4$

11. $\frac{dh}{dt} = \frac{2}{t}, \qquad h(1) = 3$

12. A ball is thrown upwards from a height of 256 ft. above the ground with an initial velocity of 96 ft/sec. (Assume that the acceleration due to gravity is 32 ft/sec^2.)

a. Determine the velocity and position of the ball above the ground for any time, t.

b. Find how high the ball goes (maximum height). Determine when it hits the ground and its velocity at that time. You should sketch a graph of the height of the ball as a function of t.

13. Gravity on the moon is about 1/6 that of Earth with $g = 1.62\text{m/sec}^2$. Suppose a lunar rock is pushed off a cliff that has a height of 324m.

a. If $h(t)$ is the height of the rock at any time t, then the rock satisfies the initial value problem:

$$h''(t) = -g, \qquad h(0) = 324 \quad \text{and} \quad v(0) = 0,$$

where $v(t)$ is the velocity of the rock. Solve this initial value problem.

b. Find when the rock hits the ground and the velocity at that instant. You should sketch a graph of the height of the rock as a function of t.

14. a. Suppose that a cat falls from the 15^{th} floor of an apartment building, 39.2 m above the ground. Let acceleration of gravity $g = 9.8$ m/sec^2, and assume that the cat satisfies Newton's law of motion for an object falling without any air resistance, so

$$\frac{d^2s}{dt^2} = g,$$

where $s(t)$ is the position of the object (with positive distance pointing down). Assume that the apartment ($s(0) = 0$) is 39.2 m above the ground and the initial velocity of the cat is zero, so $s'(0) = v(0) = 0$. Give the expressions for both the velocity and position of the cat as functions of time. Determine how long the cat falls before hitting the ground, and its velocity when it hits the ground.

b. Actually the air resistance is quite significant, and the differential equation becomes

$$\frac{dv}{dt} = g - kv,$$

where $v(t)$ is the velocity and k is the coefficient of air resistance. Let $k = 0.35$ sec^{-1} and $v(0) = 0$. Solve this differential equation and determine its velocity at $t = 3$ sec.

c. The position $s(t)$ (with $s(0) = 0$) satisfies the differential equation

$$\frac{ds}{dt} = v(t),$$

where $v(t)$ is the solution you obtained in Part b. Find $s(t)$ and determine the position at t = 3 sec.

d. The equation for the position $s(t)$ is too complicated to solve to find when the cat hits the ground. However, the numerical technique called Newton's method can give a reasonable approximation to the time that the cat hits the ground. Newton's formula for this problem is given by

$$t_{n+1} = t_n - \frac{s(t_n) - 39.2}{v(t_n)},$$

where $s(t)$ and $v(t)$ are found in Parts b and c. Let $t_0 = 3$ and perform two Newton iterates to estimate the time when the cat hits the ground. Estimate the cat's velocity when it hits the ground using the Newton approximate solution, t_2.

15. The polychlorinated biphenyls (PCBs) were key chemicals used in the electric industry to insulate transformers. They are long-lived compounds that have been linked to cancer and hormonal problems. PCBs accumulate in the fatty tissues of fish, particularly the larger predatory fish.

a. Walleye are an important game fish in the Great Lakes region. Assume that the mass of a walleye satisfies the following differential equation:

$$\frac{dw}{dt} = 0.03(10 - w), \qquad w(0) = 0,$$

where w is the mass in kg and t is the time in years. Solve this differential equation. Use the results to determine how long it takes to produce a 2 kg walleye. What is the maximum size of walleye that this equation allows? You should sketch a graph of the mass of walleye as a function of years.

b. The PCBs accumulate as the fish grows and are not removed. Assume that the intake of PCBs is proportional to the mass of the fish, so satisfies the differential equation

$$\frac{dP}{dt} = kw(t), \qquad P(0) = 0,$$

with $k = 0.3$ (mg of PCBs/kg-yr) and P being the mg of PCBs in the walleye. Solve this differential equation. Find the amount of PCBs in walleyes that are 2, 5 and 10 years old.

c. If the PCBs were uniformly spread in the fish (which they are not as they concentrate in the fatty tissues), then the concentration of PCBs, $c(t)$ (in μg/g), would be given by the formula

$$c(t) = \frac{P(t)}{w(t)}.$$

Find the mass of the fish, $w(t)$, and concentration of PCBs, $c(t)$, at times $t = 2$, 5 and 10.

16. As children age, their play activity generally decreases their exposure to lead (though initially it increases as a baby goes from being immobile through crawling). An alternative model, which shows lead concentrations in children as they age, is given below. (It is inappropriate for young children 0-2.)

a. Assume that the mass (in kg) of a girl satisfies the differential equation

$$\frac{dw}{dt} = 0.07(65 - w), \qquad w(0) = 3,$$

where t is in years. Solve this differential equation. Find the predicted masses for a girl ages 2, 4, 7, and 10. At what age does the model predict a girl reaches 50kg? What is the limiting mass for a woman based on this model? You should sketch a graph of the mass of a girl as a function of age.

b. Because of the declining exposure, the amount of lead in the body, P (in μg), satisfies the differential equation

$$\frac{dP(t)}{dt} = 100e^{-0.4t}w(t), \qquad P(0) = 0,$$

where $w(t)$ is the solution from Part a. Solve this differential equation and determine the amount of lead in a girl at girl ages 2, 4, 7, and 10.

c. If the lead is uniformly distributed in the body, then the concentration of lead in the blood, c (in μg/dl), satisfies the equation

$$c(t) = \frac{0.1P(t)}{w(t)}.$$

Find the concentration of lead in a girl ages 2, 4, 7, and 10.