Find the derivatives of the following functions:

1. $f(x) = (x^2 - 3x + 4)^4$,
2. $f(x) = \ln(22 + \sin(7x))$,
3. $f(x) = e^{-6x}(3x + 15)^2 + \ln(x^5 - 10)$,
4. $f(x) = x \sin(x^2 - 7)$,
5. $f(x) = \left(x^2 - e^{-x^3}\right)^{-1} + \dfrac{1}{x^2 + 4}$,
6. $f(x) = \left(x^4 - \cos(4x^5)\right)^6$.
7. $f(x) = \dfrac{1}{\sin^2(x^3)}$.

Find the derivative and second derivative of the functions below. Give the domain of each of the functions. Find the critical points, and determine if they are relative maxima or minima, for each graph. Find any points of inflection. Also, give the x and y-intercepts and any asymptotes if they exist. Decide if the function is odd, even or neither, and sketch the graph on separate paper.

8. $y = 2e^{-\frac{x^2}{2}}$,
9. $y = \ln(x^2 + 1)$,
10. $y = \dfrac{10x}{(1 + 0.1x)^2}$.

11. A study of American girls ages 4-13 in the 90^{th} percentile found that their height h (in cm) as a function of their age a (in years) satisfies the equation

$$h(a) = 6.44a + 82.1.$$

The same study found that their weight W (in kg) as a function of their height is given by

$$W(h) = 0.0000302h^{2.84}.$$

a. What is a rate of growth in height?

b. Write an expression for the composite function that gives the weight as a function of age, $W(a)$. Differentiate this function to find $W'(a)$ using the chain rule.

c. What is the rate of change in weight at ages 4, 8, and 13?

12. Hassell's model is often used to study populations of insects. Suppose that the updating function for the population of a species of moth P is given by

$$H(P) = \frac{5P}{(1 + 0.002P)^4}.$$

a. Find all equilibria of the model by solving the equation $H(P_e) = P_e$.

b. Differentiate this rate function, *i.e.*, find $H'(P)$.

c. Find the intercepts, all extrema, and any asymptotes for $H(P), (P > 0)$, then sketch a graph of $H(P)$.

13. The continuous logistic growth model is a very important model used in Biology. Suppose that a population of bacteria satisfies the logistic growth model

$$B(t) = \frac{100}{1 + 9e^{-0.02t}},$$

where t is in minutes and B is in thousands of bacteria/ml.

a. Compute both the first and second derivatives of $B(t)$. Give the t and B values for any points of inflection ($t \geq 0$). The population is growing most rapidly at this point of inflection. Find this most rapid rate of growth.

b. Find the B-intercept. Find any asymptotes for the function $B(t)$, then sketch its graph.

14. The growth in length of sculpin is approximated by the von Bertalanffy equation

$$L(t) = 16\left(1 - e^{-0.4t}\right),$$

where t is in years and L is in cm. An allometric measurement of sculpin shows that their weight can be approximated by the model

$$W(L) = 0.07L^3,$$

where W is in g.

a. Find the intercepts and any asymptotes for the length of a sculpin, then sketch a graph showing the length of a sculpin as it ages.

b. Create a composite function to give the weight of the sculpin as a function of its age, $W(t)$. Find the intercepts and any asymptotes for $W(t)$, then sketch a graph showing the weight of a sculpin as it ages.

c. Find the derivative of $W(t)$ using the chain rule. Also, compute the second derivative. Give both the t and W values for any points of inflection ($t > 0$). The sculpin are gaining weight most rapidly at this point of inflection. Find this most rapid rate of growth.

15. Suppose that after a burn a pioneering plant community has its biomass accumulating according to the following growth model,

$$P(t) = 20\left(1 - e^{-0.2t}\right),$$

where t is in years and P is in metric tons. The herbivores that graze on this plant community satisfy the equation

$$H(P) = 3\left(1 - e^{-0.1P}\right),$$

where H is in metric tons of the biomass of herbivores.

a. Find the intercepts and any asymptotes for the biomass of the plant community. You should make a sketch of graph showing the biomass of the plant community as time progresses on a piece of paper. Compute the derivative to determine the rate of change in biomass of the plant material. What is the rate of change in biomass at $t = 0$, 2, 10, and 20 years?

b. Create the composite function to find the biomass of the herbivores as a function of time $H(t)$. Differentiate this function, then find the rate of change in biomass of the herbivores at $t = 0$, 2, 10, and 20 years.