Spring 2012 Math 342B Take-Home Exam 1

I, _____(your name), pledge that this exam is completely my own work, and that I did not take, borrow or steal work from any other person, and that I did not allow any other person to use, have, borrow or steal portions of my work. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

1. Consider the following matrix:

$$A = \begin{pmatrix} 0 & -a & b \\ -a & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

a. Find the eigenvalues and eigenvectors for this matrix.

b. Does the set of matrices of the type above (with a and b arbitrary) form a vector space? If so, find a basis and determine its dimension. If not, explain why.

2. Calculate $\Gamma\left(-\frac{7}{2}\right)$. (Give you answer exactly.)

3. Consider the integral

$$\int_0^{\pi} \cos^6(\theta)\, d\theta.$$

Compute this integral in terms of Beta functions, Gamma functions, and an exact value.

4. Use elliptic integrals to evaluate the integral

$$\int_0^3 \frac{dx}{\sqrt{(16-x^2)(9-x^2)}}.$$

Also, give the numerical value for this integral.

5. Find the best (in the least squares sense) fifth-degree polynomial approximation to the function

$$f(x) = \sin(x)$$

over the interval $-1 < x < 1$. State your solution as a finite sum of Legendre polynomials.

6. Use Bessel functions to find the general solution to the differential equation:

$$xy'' - y' + y = 0.$$

7. Consider the differential equation given by:

$$y'' - xy' - y = 0.$$

Find the recurrence relation and two linearly independent solutions about $x_0 = 0$.

8. Show that the differential equation below has a regular singular point at $x = 0$. Give the indicial equation and recurrence relation. Determine two linearly independent solutions for $x > 0$:

$$x^2y'' + 3xy' + (1 + x)y = 0.$$