Computer Problem 3 - Fourier Integral

Consider the function

$$f(x) = \begin{cases} 0, & x < 0, \\ \pi e^{-x}, & x > 0. \end{cases}$$

The Fourier integral formula is given by

$$f(x) = \int_0^\infty [A(\omega)\cos(\omega x) + B(\omega)\sin(\omega x)]d\omega$$

1. Find the Fourier integral coefficients $A(\omega)$ and $B(\omega)$. Give the Fourier integral representation for $f(x)$.

2. Determine what the Fourier integral converges to for all values of x.

3. The truncated Fourier integral formula is given by

$$f(x) \approx \int_0^a [A(\omega)\cos(\omega x) + B(\omega)\sin(\omega x)]d\omega,$$

where a represents the truncated wave numbers. Graph the original function and the Fourier integral representation for $a = 5$, 10, 50, and 100. Show the graph for $x \in [-10, 10]$.