Monte Carlo Techniques

The Monte Carlo method (or method of statistical trials) consists of solving various problems of computational mathematics by means of some random process. This process is used for complex modeling problems which are too difficult to solve by simpler modeling techniques which can be handled analytically or via deterministic methods. The Monte Carlo method uses knowledge from past experience to assign probabilities to individual events. A set of rules for a simulation are established, then a series of simulations are performed to determine optimal solutions to the problem or identification of unknown parameters in the system.

The first example that we will consider is a based on the work of H. H. Schmitz and N. K. Kwak (*Operations Research* **20**, 1171-1180 (1970)). They used Monte Carlo methods to determine how many operating rooms were needed, how long the recovery rooms needed to be staffed, and how many beds would be needed in the recovery rooms. They derived their data from plans at Deaconesse Hospital in 1970 to expand their facilities to include 144 new beds. The question arose as to how this would effect the surgery facilities at the hospital. More specifically, how many more surgical procedures will be performed based on this increase in bed capacity and how will this effect the operating and recovery room facilities at the hospital?

The data in 1970 indicate that 42% of the patients staying at the hospital required surgery. This implies that 60 of the 144 new beds would be used primarily for surgery patients assuming that the mix of patients admitted to the hospital came in the same proportions. One could see the possibility that this assumption fails as new facilities would likely encourage more "non-essential" surgical procedures. However, it makes a reasonable first assumption for our modeling purposes. The researchers examined the past history of surgical procedures and concluded that the 60 new beds would result in 3376 new surgical cases giving the hospital a total yearly load of 9669 surgery cases. If these cases are spread evenly over the entire year, then the daily case load would be 27. (If one omitted Sundays and 10 holidays, then this case load would increase to 32 cases per day.)

To answer our questions about details on how many operating rooms are needed, we need more information on the types of procedures performed and the length of time of the operations. Also, we need some idea of how these operations effect the recovery room facilities. With this information, can the hospital determine how it should schedule its surgeries and staff its recovery rooms? An analysis was made on the surgeries performed on 445 patients, and it was found that the length of stay in the operating room is exponentially distributed with an interarrival time mean of 1.03 hours. This indicates that if 4 operating rooms were used then operations would occupy the operating rooms for about 7 hours per day, while 5 operating rooms would cause this to drop to about 6 hours per day. Below we will show how a Monte Carlo simulation can be used to show more about the variation in use of the operating room facilities.

A survey of types of surgery performed gave the results summarized in Table 1. This table shows the length of the surgery, the relative frequency, and the random numbers associated with each type of surgery.

Schmitz and Kwak assume that if the procedure lasts from 0.0-0.5, then they use 0.5 hours. Other procedures are assumed to last the length of time which matches the midpoint of the interval with the last case assumed to last exactly 4 hours. Notice that this assumption does violate the exponential form that they found holds for surgical procedures. A more complicated simulation could be performed by subdividing the random numbers to more closely match the exponential form of the distribution function for time of surgery.

Below we list the rules that are applied in the Monte Carlo simulation.

1. The daily case load is assumed to be fixed at 27 cases.

Type of Surgery	Time Interval	Relative Frequency	Random Numbers
Ear-Nose-Throat	0.0-0.5	15.8	000-157
Urology (To RR)	0.0-0.5	8.4	158-241
Urology (No RR)	0.0-0.5	8.5	242-326
Opthalmology (No RR)	0.0-0.5	5.8	327-384
Other Surgery	0.5-1.0	23.6	385-620
Other Surgery	1.0-1.5	14.6	621-766
Other Surgery	1.5-2.0	9.0	767-856
Other Surgery	2.0-2.5	5.5	857-911
Other Surgery	2.5-3.0	3.4	912-945
Other Surgery	3.0-3.5	2.1	946-966
Other Surgery	3.5-4.0	1.3	967-979
Other Surgery	>4.0	2.0	980-999

2. Random numbers are generated independently for each day.
3. All ENT, urology, and ophthamology cases last 0.5 hours.
4. Half the urology cases and all ophthamology cases do not go to the recovery room.
5. All ENT and the other half of urology cases go to the recovery room and are assumed to stay for 1.5 hours.
6. Any operation over 0.5 hours is considered major and requires 3 hours in the recovery room.
7. Surgery begins at 7:30 a.m.
8. Preparation time is .25 hours in the operating room.
9. It takes .08 hours to transport patients from operating room to the recovery room.
10. It takes .25 hours to prepare the recovery room for the next occupant.
11. Operating rooms are used continuously as need arises with the first one vacated being the first one used.
12. The first vacated recovery bed is the first one filled as needed.
13. If no bed available in the recovery room, then a new one is created.

The attached sheet shows the results of one simulation. Four days were simulated with 5 operating rooms. The attached simulation shows that surgery ended at 14:24, while the recovery room was cleared by 17:44. The latest surgery lasted to 17:30 for the 4 days with the recovery room clearing by 20:36. This was the optimal solution found by Schmitz and Kwak. The use of 4 operating rooms made for days going too long, while 6 operating rooms often completed all surgery before noon.

Problem

MC1. Perform a two simulations each for 4 and 5 operating rooms using a patient load of 32 patients per day (which would be the case load with Sundays and some holidays off). You can obtain your random numbers from any source that you wish. The computer has several good random number generators which you may wish to use.

Patient #	Random #	Length of Operation	Time Begins	Time Ends	Operating Room #	Recovery Y/N	Recovery Begins	Recovery Ends	RR Bed #	RR Bed Available
1	889	2.25	7.50	9.75	1	Y	9.83	12.83	7	13.08
2	396	0.75	7.50	8.25	2	Y	8.33	11.33	1	11.58
3	358	0.50	7.50	8.00	3	N	-	-	-	-
4	715	1.25	7.50	8.75	4	Y	8.83	11.83	3	12.08
5	502	0.75	7.50	8.25	5	Y	8.33	11.33	2	11.58
6	068	0.50	8.25	8.75	3	Y	8.83	10.33	4	10.58
7	604	0.75	8.50	9.25	2	Y	9.33	12.33	5	12.58
8	270	0.50	8.50	9.00	5	N	-	-	-	-
9	228	0.50	9.00	9.50	4	Y	9.58	11.08	6	11.33
10	782	1.75	9.00	10.75	3	Y	10.83	13.83	4	14.08
11	379	0.50	9.25	9.75	5	N	-	-	-	-
12	093	0.50	9.50	10.00	2	Y	10.08	11.58	5	11.83
13	011	0.50	9.75	10.25	4	Y	10.33	11.83	9	12.08
14	648	1.25	10.00	11.25	1	Y	11.33	14.33	6	14.58
15	527	0.75	10.00	10.75	5	Y	10.83	13.83	10	14.08
16	987	4.15	10.25	14.40	2	Y	14.48	17.48	5	17.73
17	214	0.50	10.50	11.00	4	Y	11.08	12.58	11	12.83
18	474	0.75	11.00	11.75	3	Y	11.83	14.83	2	15.08
19	238	0.50	11.00	11.50	5	Y	11.58	13.08	1	13.33
20	045	0.50	11.25	11.75	4	Y	11.83	13.33	5	13.58
21	408	0.75	11.50	12.25	1	Y	12.33	15.33	3	15.58
22	116	0.50	11.75	12.25	5	Y	12.33	13.83	9	14.08
23	209	0.50	12.00	12.50	3	Y	12.58	14.08	5	14.33
24	048	0.50	12.00	12.50	4	Y	12.58	14.08	12	14.33
25	393	0.75	12.50	13.25	1	Y	13.33	16.33	11	16.58
26	550	0.75	12.50	13.25	5	Y	13.33	16.33	7	16.58
27	306	0.50	12.75	13.25	3	N	-	-	-	-