1. a. The payment is given by

$$M = \frac{0.06}{12}\frac{13000(1.005)^{60}}{(1.005)^{60}-1} = 251.3264.$$

The total cost is given by

$$T = 60M + 1000 = 16,079.585.$$

b. A discrete dynamical model is given by

$$P_{n+1} = 1.005P_n - 500.$$

From Excel, we find that after 27 months there is \$458.79 remaining. Thus, we pay off the car in 28 months, and the total cost is

$$T = 27(500) + 1000 + (1.005)458.70 = 14,961.08.$$

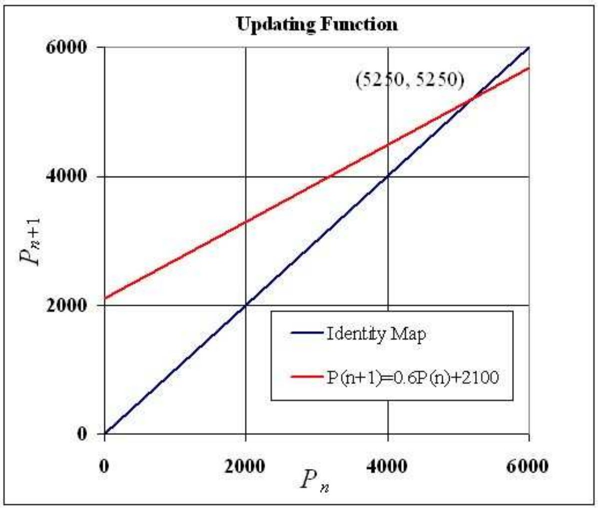
2. a. The data on the flying insect population gives the equations

$$\begin{aligned} 9300 &= 12000r + \mu, \\ 7680 &= 9300r + \mu, \end{aligned}$$

which solved simultaneously gives $r = 0.6$ and $\mu = 2100$. With this information, we find the populations in 1993 and 1994 given by $P_3 = 6708$ and $P_4 = 6125$, respectively.

b. Solving for the equilibrium, we find the only equilibrium is $P_e = 5250$. This equilibrium is stable as the derivative of the updating function is less than 1. Ultimately, this model predicts that the population of flying insects will stabilize at 5250.

c. Below is a graph of the updating function and the identity map.



3. a. Since the solution to the Malthusian growth model is $P_n = 53.4(1+r)^n$, then $123.0 = 53.4(1+r)^3$ or $r = 0.32065$. It follows that the doubling time for Brazil's population is 2.162 decades or 21.62 years.

b. From the Malthusian growth model, the population in 2000 is estimated to be

$$P_5 = 53.4(1.32065)^5 = 214.53\,\mathrm{M}.$$

The error from the actual census value is

$$Error = 100\frac{214.53 - 175.6}{175.6} = 22.17\%.$$

c. From the logistic growth model, the estimated populations in 1960, 1970, and 1980 are given by $P_1 = 72.79$ M, $P_2 = 96.41$ M, and $P_3 = 123.13$ M, respectively. The error from the actual census value in 1980 is

$$Error = 100\frac{123.13 - 123.0}{123.0} = 0.1083\%.$$

d. The equilibria for the logistic growth model are $P_e = 0$ and 235.0 M. The derivative of $F(P)$ is

$$F'(P) = 1.47 - 0.004\,P.$$

At the equilibrium $P_e = 0$, $F'(0) = 1.47$, which says that this equilibrium point is unstable with solutions monotonically growing away from the equilibrium. At the equilibrium $P_e = 235.0$, $F'(235.0) = 0.53$, which says that this equilibrium point is stable with solutions monotonically approaching the equilibrium. This model predicts that the population of Brazil will stabilize at its carrying capacity of 235.0 M people.

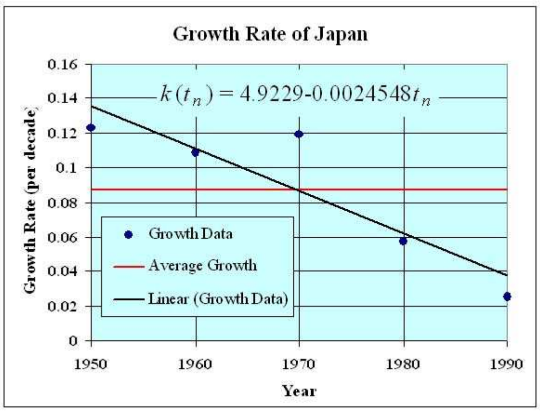
4. a. The growth rate (per decade) for Japan is given by The average growth rate is $r = 0.086861$

Year	Growth
1950	0.12266
1960	0.10894
1970	0.11951
1980	0.057615
1990	0.025579

per decade. The best linear fit to the growth data is given by

$$k(t_n) = 4.9229 - 0.0024548t_n,$$

where $t_n = 1950 + 10n$.



b. The general Discrete Malthusian growth model for Japan is given by

$$P_n = 83.81(1.086861)^n.$$

This model predicts the populations in 2000, 2020, and 2050 to be 127.11 M, 150.15 M, and 192.77 M, respectively.

c. The revised growth model is given by

$$P_{n+1} = (5.9229 - 0.0024548t_n)P_n,$$

where $t_n = 1950 + 10n$. This nonautonomous discrete dynamical model predicts the population of Japan in 2000, 2020, and 2050 to be 126.83 M, 127.07 M, and 105.36 M, respectively.

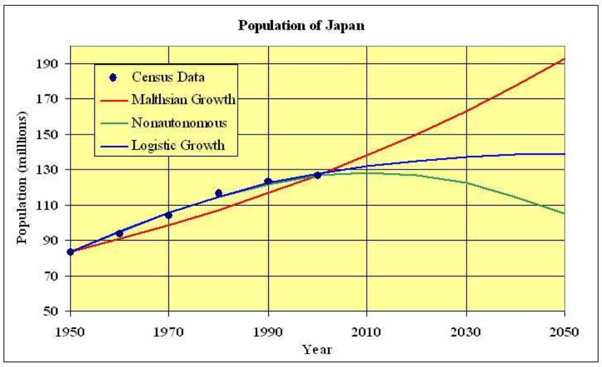
d. Solving $k(t) = 0$ gives the growth rate of Japan to be zero when $t = 2005.4$. Since this growth rate corresponds to the beginning of the decade (by choice of t_n), it follows that the growth rate is zero between 2005 and 2015. Hence, the maximum population is approximately the population in 2010, which is 128.5 M.

e. Using a least squares best fit to the data, the best logistic growth model is given by

$$P_{n+1} = P_n + 0.33906P_n\left(1 - \frac{P_n}{141.57}\right) = F(P_n) \quad \text{with} \quad P_0 = 83.81.$$

This model predicts the populations in 2000, 2020, and 2050 to be 127.95 M, 135.12 M, and 139.61 M, respectively. The equilibria are $P_e = 0$ and 141.57. The derivative of the updating function is $F'(P) = 1.33906 - 0.004790P$. It follows that the extinction equilibrium $P_e = 0$ is unstable with solutions monotonically growing away as $F'(0) = 1.33906$, while the equilibrium at the carrying capacity $P_e = 141.57$ is stable with solutions monotonically approaching this equilibrium as $F'(141.57) = 0.66094$. This model predicts that the population of Japan will grow to the carrying capacity 141.57 M and level off over time.

f. Below is a graph of the three models with the census data. The sum of square errors for

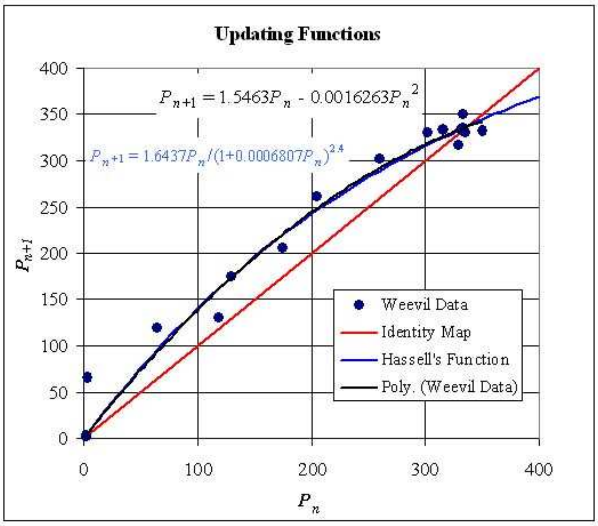


the Malthusian growth model is 165.92, the sum of square errors for the nonautonomous growth model is 8.443, and the sum of square errors for the logistic growth model is 10.716. Thus, the nonautonomous growth model fits the data best, but it is only marginally better than the logistic growth model. In 2000, the population of Japan was 126.7 M. The Malthusian growth model predicted 127.11 M, which gives a 0.32% error. The nonautonomous growth model predicted 126.83 M, which gives a 0.103% error. The logistic growth model predicted 127.95 M, which gives a 0.9895% error. The Malthusian growth model uses a simple exponential growth of the population, which does not reflect the actual growth very well. The nonautonomous and logistic growth models show trends that very closely match the census data for the years 1950 to 2000, but then the nonautonomous model reaches a maximum population shortly after 2000, then begins declining, while the logistic growth model continues to increase with slowing growth to a carrying capacity of 141.57 M. It is hard to predict which of the later two models is more accurate in its predictions though it is likely that the actual trend will lie between these models. The Malthusian growth model is too simple to accurately predict human populations. The nonautonomous and logistic growth models are relatively simple, which make them excellent models to match limited data as was presented. The logistic growth is very similar to what is seen in many animal populations, while the nonautonomous model better reflects changing human society with a time varying growth rate. All of the models have weaknesses that include the lack of age-structure in the populations and don't account for immigration and emigration, which can be significant.

5. a. From the data, the best discrete logistic growth model for the adult population of *Rhizopertha dominica*, the American wheat weevil, P_n, can be written

$$P_{n+1} = f(P_n) = 1.5463P_n - 0.0016263P_n^2.$$

Below is a graph both $f(P)$ and the data along with the identity map, $P_{n+1} = P_n$. The graph also includes the updating function for Hassell's model in Part c.



b. The equilibria for this logistic growth model are $P_e = 0$ and 335.92. The derivative for this updating function is

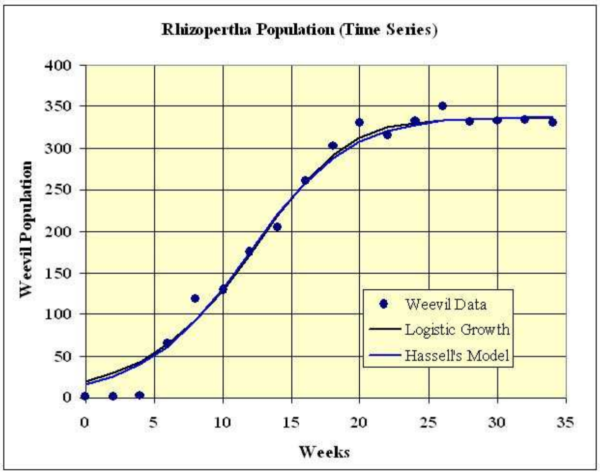
$$f'(p) = 1.5463 - 0.0032526p.$$

It follows that the extinction equilibrium $P_e = 0$ is unstable with solutions monotonically growing away as $f'(0) = 1.5463$, while the equilibrium at the carrying capacity $P_e = 335.92$ is stable with solutions monotonically approaching this equilibrium as $f'(335.92) = 0.4537$. The logistic growth model is simulated, and the best fit is found by adjusting P_0. The best fit gives $P_0 = 18.984$ with a sum of square errors being 4334.75. The simulation shows that the model simulates the data fairly well after the initial 4 weeks. Over large time, this population will approach the carrying capacity of $P_e = 335.92$. Note that the graph also includes the simulation for Hassell's model in Part d.

c. From the data, the best fitting Hassell's model with $c = 2.4$ is given by

$$P_{n+1} = H(P_n) = \frac{1.6437P_n}{(1 + 0.00068072P_n)^{2.4}}.$$

Thus, we have $a = 1.6437$ and $b = 0.00068072$ with the sum of square errors being 6700.25. The graph of this updating function is seen along with the logistic updating function above. Note that the two graphs are very similar.



d. The equilibria for Hassell's model are $P_e = 0$ and 337.96. The derivative for this updating function is

$$H'(p) = 1.6437\ (1 + 0.00068072\, p)^{-2.4} - 0.002685358714\, p\, (1 + 0.00068072\, p)^{-3.4}\,.$$

It follows that the extinction equilibrium $P_e = 0$ is unstable with solutions monotonically growing away as $H'(0) = 1.6437$, while the equilibrium at the carrying capacity $P_e = 337.96$ is stable with solutions monotonically approaching this equilibrium as $H'(337.96) = 0.5511$. Hassell's model is simulated, and the best fit is found by adjusting P_0. The best fit gives $P_0 = 15.964$ with a sum of square errors being 4192.235. The simulation shows that the model simulates the data fairly well after the initial 4 weeks, and closely matches the simulation of the logistic growth model. Over large time, this population will approach the carrying capacity of $P_e = 337.96$. (The graph is shown above with the logistic growth model in Part b.) Both models have two free parameters, and the sum of square errors are very similar. Thus, each of these models does about the same simulating the given data with very little difference observable from the simulations (or updating functions). The primary difference is that for very large populations, the logistic updating function becomes negative, while the Hassell's updating function remains positive.