Spring 2007 Math 336 Name________________________________

I, ________________________________(your name), pledge that this exam is completely my own work, and that I did not take, borrow or steal work from any other person, and that I did not allow any other person to use, have, borrow or steal portions of my work. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

Give all answers to at least **4 significant figures**. Be sure to include appropriate **units** for each of your answers.

1. a. Suppose that you purchase a car for \$14,000 by making a down payment of \$1000 and taking a 5 year loan at 6% interest (annual rate). Find the monthly payment on this loan and how much you spend over the 5 years of your loan.

b. Suppose that you can spend \$500 per month. How long does it take for you to pay off this car and what is the total that you spend on this car using this alternate payment schedule?

2. Below are data on the population of a species of flying insect that inhabits an island and breeds annually (then dies). If its offspring have a survival rate r, and there is a net (constant) influx of new insects from surrounding islands entering at a rate μ, then the population model has the form

$$P_{n+1} = rP_n + \mu.$$

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of the flying insects in 1993 and 1994.

Year	Insects
1990	12,000
1991	9,300
1992	7,680

b. Find all equilibria for this model. What is the stability of the equilibria? Give a reason for your answer. What does this model predict will ultimately happen to the population of flying insects?

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Label both curves clearly on your graph. Determine all points of intersection.

3. a. The population of Brazil in 1950 was about 53.4 million, while in 1980, it was about 123.0 million. Assume that the population is growing according to the discrete Malthusian growth equation

$$P_{n+1} = (1 + r)P_n, \quad \text{with} \quad P_0 = 53.4,$$

where P_0 is the population in 1950 and n is in decades. Use the population in 1980 to find the value of r. Find how long it would take for this population to double.

b. Estimate the population in 2000 based on the Malthusian growth model. Given that the population in 2000 was 175.6 million, find the percent error between the actual and predicted values.

c. A better model fitting the census data for Brazil is a logistic growth model given by

$$P_{n+1} = F(P_n) = 1.47P_n - 0.002P_n^2,$$

where again n is in decades after 1950. Estimate the populations in 1960, 1970, and 1980 by computing P_1, P_2, and P_3, where $P_0 = 53.4$. Find the percent error between the actual and predicted values in 1980.

d. Find the equilibria for this logistic growth model. Calculate the derivative of $F(P)$ and evaluate it at all equilibria. What does this value say about the behavior of the solution near these equilibria? What does this model predict will eventually happen to the population of Brazil?

4. Using data from the U. S. census bureau, the table below presents the population (in millions) for Japan.

Year	Population
1950	83.81
1960	94.09
1970	104.34
1980	116.81
1990	123.54
2000	126.70

a. Find the growth rate for each decade with the data above by dividing the population from one decade by the population of the previous decade and subtracting 1 from this ratio. Associate each growth rate with the earlier of the two census dates. Determine the average (mean) growth rate, r, from the data above. Let $t_n = 1950 + 10n$ so that t_n corresponds to the actual dates.

$$k(t_n) = a + bt_n$$

through the growth data. Graph the constant function r, $k(t_n)$, and the data as a function of t_n over the period of the census data.

b. The Discrete Malthusian growth model is given by

$$P_{n+1} = (1 + r)P_n.$$

where r is computed in Part a. and P_0 is the population in 1950. Write the general solution to this model, where n is in decades after 1950. Use the model to predict the population in 2000, 2020, and 2050.

c. The revised growth model is given by

$$P_{n+1} = (1 + k(t_n))P_n.$$

where $k(t_n)$ is computed in Part a. and P_0 is again the population in 1950. Simulate this nonautonomous discrete dynamical model from 1950 to 2050. Use the model to predict the population of Japan in 2000, 2020, and 2050.

d. The growth rate of the nonautonomous dynamical model goes to zero during this century for Japan. At this time, this model predicts that the population will reach its maximum and start declining. Use the growth rate $k(t)$ to find when this model predicts a maximum population, then estimate what that maximum population will be.

e. Use the data above to find the best discrete logistic growth model fit for the population of Japan. (P_0 is again the population in 1950, so only vary the growth rate and carrying capacity when doing the least squares best fit.) What does this model predict for the population of Japan in 2000, 2020, and 2050? Find all equilibria of this model and discuss the stability of these equilibria (include the values of the derivatives at the equilibria). What does this model predict will happen over a long period of time for Japan's population?

f. Graph all three models with the data from 1950 to 2050. (Use points for census data and lines for all theoretical models in your graphs.) Give the sum of square errors for all three models. From the sum of square errors, which model matches the data best? Find the percent error of each of the models in 2000. Briefly discuss how well these models predict the population over this period. List some strengths and weaknesses of each of the models and how you might obtain a better means of predicting the population.

5. A. C. Crombie studied *Rhizopertha dominica*, the American wheat weevil, with an almost constant nutrient supply (maintained 10 g of cracked wheat weekly). These conditions match the assumptions of the discrete logistic model. The data below show the adult population of *Rhizopertha* from Crombie's study (with some minor modifications to fill in uncollected data and an initial shift of one week).

Week	Adults	Week	Adults
0	2	18	302
2	2	20	330
4	3	22	316
6	65	24	333
8	119	26	350
10	130	28	332
12	175	30	333
14	205	32	335
16	261	34	330

a. The discrete logistic growth model for the adult population P_n can be written

$$P_{n+1} = f(P_n) = rP_n - mP_n^2,$$

where the constants r and m must be determined from the data. In this part of the problem, you want to find the logistic growth updating function by graphing P_{n+1} vs. P_n, which you can do by entering the adult population data from times 2–34 for P_{n+1} and times 0–32 for P_n. Find the appropriate constants r and m by fitting the best quadratic (of the appropriate form) to these data. Graph both $f(P)$ and the data along with the identity map, $P_{n+1} = P_n$.

b. Find the equilibria for this model. Write the derivative of the updating function. Discuss the behavior of the model near its equilibria. Use the model found above to simulate the data. Find the best fitting model to the data by using the model found in Part a and adjusting the initial condition, P_0, to give the least squares best fit to the data. Write both the sum of square errors and P_0. Graph this simulation and the data (adult population vs. time). Discuss how well your simulation matches the data in the table. What do you predict will happen to the adult American wheat weevil population for large times (assuming experimental conditions continue)?

c. Another common population model is Hassell's model, which is given by

$$P_{n+1} = H(P_n) = \frac{aP_n}{(1 + bP_n)^c},$$

where a, b, and c are constants to be determined. Suppose that it is found that for this type of beetle, the best value of $c = 2.4$. Find the least squares best fit of the Hassell's updating function to the given data by varying a and b. Give the values of a and b along with the sum of square errors for this fit. Once again graph both $H(P)$ and the data along with the identity map, $P_{n+1} = P_n$. How does this updating function compare to the one given in Part a?

d. Find the equilibria for Hassell's model. Write the derivative of this updating function. Discuss the behavior of this model near its equilibria. Use Hassell's model to simulate the data. Find the best fitting model to the data by using the model found in Part c and adjusting the initial condition, P_0, to give the least squares best fit to the data. Write both the sum of square errors and P_0. Graph this simulation and the data (adult population vs. time). Discuss how well your simulation matches the data in the table. Discuss the similarities and differences that you observe between these models and how well they work for this experimental situation.