Spring 2007 Math 336 Name Key

Give all answers to at least **4 significant figures**. Be sure to include appropriate **units** for each of your answers. Partial credit may be awarded for showing correct formulae or describing how you obtained your answer on the computer.

1. a. Suppose that you purchase a car for $16,000 by making a down payment of $1500 and taking a 4 year loan at 8% interest (annual rate). Find the monthly payment on this loan and how much you spend over the 4 years of your loan for the car.

$$M = \frac{0.08}{12} \frac{14500\left(1+0.08/12\right)^{48}}{\left(1+\frac{0.08}{12}\right)^{48}-1} = \$353.99$$

$353.99 \times 48 + 1500$

10 Monthly Payment = $353.99 Total Cost of Car = $18,491.52

b. Suppose that you can spend $600 per month. How long does it take for you to pay off this car and what is the total that you spend on this car using this alternate payment schedule?

In Excel, start $14,500 $P_{n+1} = \left(1+\frac{0.08}{12}\right)P_n - 600$

26 months to $262.32

$1500 + 26 \times 600 + 262.32\left(1+\frac{0.08}{12}\right)$

10 Length of Loan = 26 months Total Cost of Car = $17,364.07

27 months with last reduced payment.

20

2. a. The population of the U. S. in 1900 was about 76.0 million. In 1990, it was about 248.7 million. Assume that the population is growing according to the logistic growth model given by

$$P_{n+1} = F(P_n) = 1.21P_n - 0.000467P_n^2,$$

where again n is in decades after 1900. Estimate the populations in 1920, 1950, and 1990 by computing P_2, P_5, and P_9, where $P_0 = 76.0$. Find the percent error between the actual and predicted values in 1990.

6 $P_2 =$ 104.29 and $P_5 =$ 159.91 and $P_9 =$ 251.30

3 Percent Error = $\frac{251.3 - 248.7}{248.7} \times 100 = 1.046\%$

b. Find the equilibria for this logistic growth model. Calculate the derivative of $F(P)$ and evaluate it at all equilibria. What does this value say about the behavior of the solution near these equilibria? What does this model predict will eventually happen to the population of the U. S.?

2 $F'(P) =$ $1.21 - 0.000934P$

4 $P_{1e} =$ 0 $F'(P_{1e}) =$ 1.21

2 **Stable** or **Unstable** **Monotonic** or **Oscillatory**

4 $P_{2e} =$ 449.68 $F'(P_{2e}) =$ 0.79

2 **Stable** or **Unstable** **Monotonic** or **Oscillatory**

2 What eventually happens to U. S. population according to this model?

Model predicts $\lim_{t \to \infty} P(t) = 449.7$ million, the carrying capacity

25

3. Using data from the U. S. census bureau, the table below presents the population (in millions) for Mexico.

Year	Population	Year	Population	Year	Population
1950	28.49	1970	52.78	1990	84.91
1960	38.58	1980	68.34	2000	99.93

a. Find the growth rate for each decade with the data above by dividing the population from one decade by the population of the previous decade and subtracting 1 from this ratio. Determine the average (mean) growth rate, r, from the data above. Let $t_n = 1950 + 10n$ so that t_n corresponds to the actual dates.

$$k(t_n) = a + bt_n$$

through the growth data.

6 $r =$ 0.2873 $k(t_n) =$ 9.7459 − 0.0048013 t

b. The Discrete Malthusian growth model is given by

$$P_{n+1} = (1 + r)P_n.$$

where r is computed in Part a and P_0 is the population in 1950. Write the general solution to this model, where n is in decades after 1950. Find how long this model predicts for Mexico's population to double. Use the model to predict the population in 2000 and 2050.

5 $P_n =$ $28.49(1.2873)^n$ Doubling time = 27.45 years

4 $P_5 =$ 100.71 M $P_{10} =$ 356.03 M

c. The revised growth model is given by

$$P_{n+1} = (1 + k(t_n))P_n.$$

where $k(t_n)$ is computed in Part a and P_0 is again the population in 1950. Simulate this nonautonomous discrete dynamical model from 1950 to 2050. Use the model to predict the population of Mexico in 2000, 2020, and 2050.

6 $P_5 =$ 100.03 M $P_7 =$ 125.26 M $P_{10} =$ 124.70

d. The growth rate of the nonautonomous dynamical model goes to zero during this century for Mexico. At this time, this model predicts that the population will reach its maximum and start declining. Use the growth rate $k(t)$ to find when this model predicts a maximum population, then estimate what that maximum population will be. Solve $k(t) = 0 \Rightarrow t = 2029.85$

4 $t_{max} =$ 2030 $P(t_{max}) \simeq$ 131.18 M

25

4. Fish that spawn seasonally have been shown to satisfy Ricker's model. Suppose that a newly stocked lake is shown to have have the following populations of a particular fish (in thousands) over the next 9 years.

Year	Fish	Year	Fish	Year	Fish
0	2	3	28	6	227
1	5	4	58	7	319
2	12	5	121	8	362

a. Ricker's model is given by

$$P_{n+1} = R(P_n) = aP_n\, e^{-bP_n},$$

where a and b are constants to be determined. Find the least squares best fit of the Ricker's updating function to the given data by varying a and b. (Reasonable initial guesses are $a = 2$ and $b = 0.002$.) Give the values of a and b along with the sum of square errors for this fit.

9 $a =$ 2.44555 $b =$ 0.0024095 Sum of square errors = 82.16

b. Find the equilibria for Ricker's model. Write the derivative of this updating function. Discuss the behavior of this model near its equilibria.

3 $R'(P) = 2.44555\, e^{-0.0024095P}(1 - 0.0024095P)$

4 $P_{1e} =$ 0 $R'(P_{1e}) =$ 2.44555

2 **Stable** or **Unstable** **Monotonic** or **Oscillatory**

4 $P_{2e} =$ 371.14 $R'(P_{2e}) =$ 0.1057

2 **Stable** or **Unstable** **Monotonic** or **Oscillatory**

c. Use Ricker's model to simulate the data. Find the best fitting model to the data by using the model found in Part b and adjusting the initial condition, P_0, to give the least squares best fit to the data. Write both the sum of square errors and P_0.

6 $P_0 =$ 1.8039 Sum of square errors = 23.397

30