1. a. From the breathing model, $c_{n+1} = (1-q)c_n + q\gamma$ and the data $c_0 = 400$, $c_1 = 352$, and $c_2 = 310$, we find the constants q and γ by substitution and the simultaneous solution of two equations and two unknowns. We have

$$352 = 400(1-q) + q\gamma \qquad \text{and} \qquad 310 = 352(1-q) + q\gamma.$$

Subtracting the second equation from the first gives $42 = 48(1-q)$ or $1 - q = \frac{42}{48} = \frac{7}{8}$. Thus, $q = \frac{1}{8}$. This value is substituted into the first equation above to give $352 = 400\frac{7}{8} + \frac{1}{8}\gamma$, which gives $\gamma = 16$.

Thus, the model becomes $c_{n+1} = \frac{7}{8}c_n + 2$, and the next 2 breaths satisfy

$$\begin{aligned} c_3 &= \frac{7}{8}310 + 2 = 273.25 \\ c_4 &= \frac{7}{8}273.25 + 2 = 241.1 \end{aligned}$$

b. At the equilibria, $c_e = \frac{7}{8}c_e + 2$, so $\frac{1}{8}c_e = 2$ or $c_e = 16$, which is the value of γ as expected. This equilibrium is stable.

c. The graph of the updating function and identity map, $c_{n+1} = c_n$, are shown below. The only point of intersection occurs at the equilibrium, γ found above.

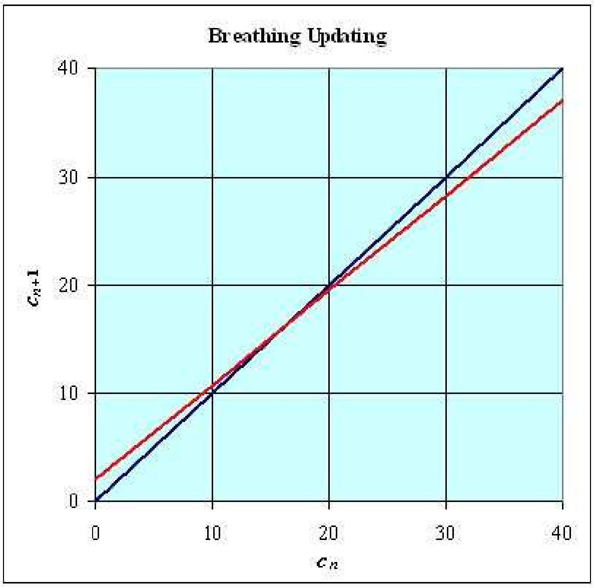


Figure 1: The identity map and the updating function intersect at the equilibria, (16, 16).

2. a. The populations for the first 5 days are $P_1 = 43,200$, $P_2 = 46,224$, $P_3 = 48,997$, $P_4 = 51,447$, $P_5 = 53,505$.

b. Growth rate is zero at $t = 8$ days with $P_8 = 56,775$.

c. One iterates until the population drops below one individual in which case the answer is $t = 52$ days.

3. a. The Malthusian growth model estimates Italy's populations in 1990 and 2000 as 60.0 and 63.6 million people. The doubling time for this model would be 117 years.

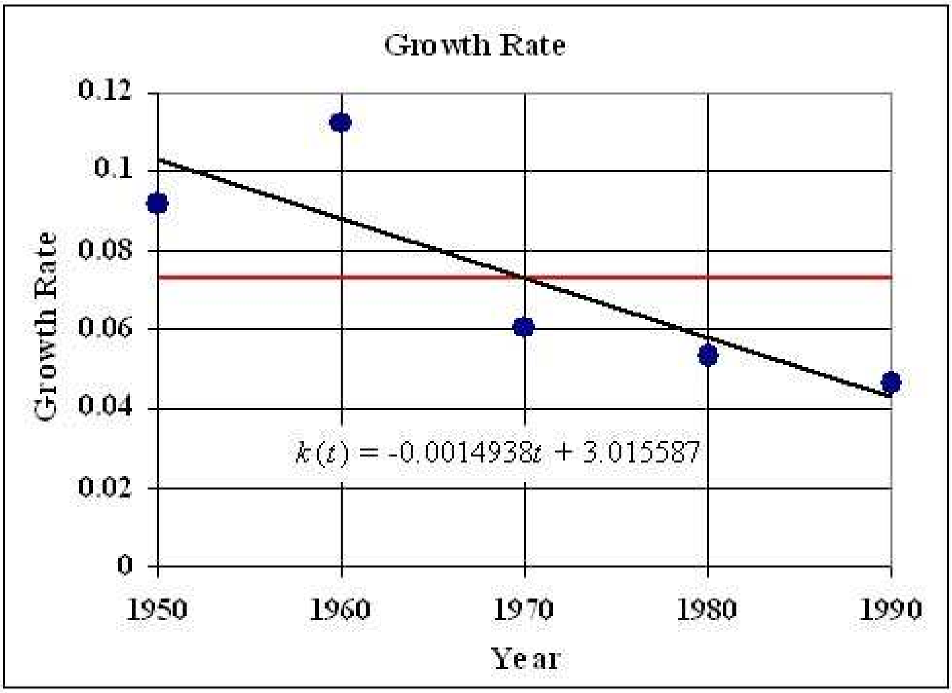
b. The Nonautonomous Malthusian growth model predicts Italy's populations in 1990 and 2000 as 58.4 and 59.4 million people. The growth term $k(t_n)$ drops to zero near 1999, so the population would level off then.

c. Both predicted values are high with the errors being 2.9% in 1990 and 2.5% in 2000. The census data are clearly leveling off now, which is consistent with the predicted leveling off in 1999.

4. a. The average growth rate is $r = 0.07287$. From Excel's Trendline, the best straight line satisfies

$$k(t) = 3.015587 - 0.0014938t,$$

where t is the earlier of the dates from the census data. Below is a graph of the growth rate from the census data with the average and best straight line fit.



b. The Discrete Malthusian growth model is given by

$$P_{n+1} = (1.07287)P_n,$$

which has a solution given by

$$P_n = 41.83(1.07287)^n.$$

This model predicts the populations to be 68.44 and 84.52 million in 2020 and 2050, respectively.

c. The nonautonomous Malthusian growth model is given by

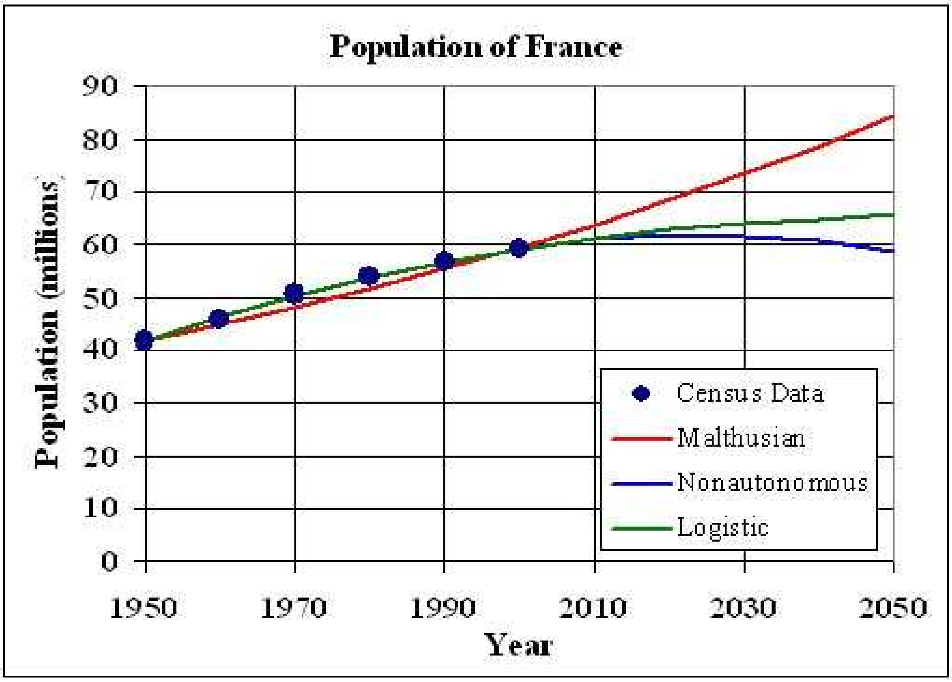
$$P_{n+1} = (4.015587 - 0.0014938t_n)P_n.$$

where $t_n = 1950 + 10n$. Simulations of this model are in the table below. This model predicts the population to be 61.84 and 58.76 million in 2020 and 2050.

d. Below is a table listing the date, the population data, the predicted values from all the Malthusian growth model, the nonautonomous dynamical model, and logistic growth model along with the percent error between the actual population and each of the predicted populations from the models from 1950 to 2000. The maximum error for the Malthusian growth model is -5.2% in 1970, while the maximum error for the nonautonomous Malthusian growth model is only -1.22% also in 1970. The logistic growth model has a maximum error of 1.3% in 1960.

Year	Census	Malthusian	% err	Nonauto	% err	Logistic	% err
1950	41.83	41.83	0	41.83	0	41.83	0
1960	45.67	44.88	-1.73	46.12	1.00	46.26	1.30
1970	50.79	48.15	-5.20	50.17	-1.22	50.31	-0.94
1980	53.87	51.66	-4.11	53.82	-0.08	53.87	0
1990	56.74	55.42	-2.32	56.94	0.35	56.88	0.25
2000	59.38	59.46	0.13	59.38	0.01	59.35	-0.06
2010		63.79		61.05		61.31	
2020		68.44		61.84		62.84	
2030		73.43		61.72		64.01	
2040		78.78		60.69		64.88	
2050		84.52		58.76		65.54	

Below is a graph of the census data and all the models used to predict the population from 1950 to 2050. The Malthusian growth model fits the data the least well, but then it is a very simple model with only the average growth parameter r. Both the logistic and nonautonomous growth models fit the data extremely well with very similar errors. It would be almost impossible to choose between these models for the better prediction without more demographic data such as age structure. They both have the same number of parameters, so are equally simple in design. Age-structure and immigration rates could be added to the observed trends in these models to make them better predictors of the future population.



e. The growth rate of the nonautonomous dynamical model goes to zero around 2019. Since the growth rate applies for the decade following its calculation, this model implies equal populations in 2019 and 2029. The maximum population predicted by this model for France would occur around the midpoint of that decade in 2024 with an estimated population of approximately 61.9 million people.

f. From the data, we find the best discrete logistic growth model has the parameters $r = 0.279975$ and $M = 67.304$ (assuming $P_0 = 41.83$ million), so the model is given by

$$P_{n+1} = P_n + 0.279975P_n\left(1 - \frac{P_n}{67.304}\right) = F(P_n).$$

This model predicts that the population in 2050 will be 65.54 million people, and that the population will level off at a carrying capacity of 67.3 million people. The sum of square errors for the logistic growth model is 0.6002, while the sum of square errors for the nonautonomous Malthusian growth model is 0.6307, suggesting negligible difference between the models. The equilibria for the logistic growth model are $P_e = 0$, which is unstable with $F'(0) = 1.28 > 1$, and $P_e = 67.304$, which is stable with $F'(67.304) = 0.72 < 1$.